A TOLERANCE ALLOCATION FRAMEWORK USING FUZZY COMPREHENSIVE EVALUATION AND DECISION SUPPORT PROCESSES

A Thesis
Presented to
The Academic Faculty

By

Abhishek Kumar

In Partial Fulfillment
Of the Requirements for the Degree
Master of Science in Mechanical Engineering

Georgia Institute of Technology

December, 2010

A TOLERANCE ALLOCATION FRAMEWORK USING FUZZY COMPREHENSIVE EVALUATION AND DECISION SUPPORT PROCESSES

Approved by:

Dr. Seung-Kyum Choi, Chair
School of Mechanical Engineering
Georgia Institute of Technology

Dr. Dirk Schaefer
School of Mechanical Engineering
Georgia Institute of Technology

Dr. Rafi Muhanna
School of Civil and Environmental Engineering
Georgia Institute of Technology

Date Approved: July 23, 2010

ACKNOWLEDGEMENTS

I would like to take this opportunity to thank all those who have helped me out in the journey towards the successful completion of my thesis. Firstly, I would like to thank my advisor, Dr. Seung-Kyum Choi, without whom I would not have been able to complete my thesis. He has guided me by providing constant support and feedback through every step of the way. Through his insistence on high quality research, results and presentation, he has helped me prepare a better thesis and a better future career. I would also like to thank the faculty members, Dr. Dirk Schaefer and Dr. Rafi Muhanna, for serving as my thesis committee members. Their time and commitment has helped improve the quality of my thesis.

Next, I would like to thank my family for all their love, support and guidance they have given me throughout my graduate career. My parents, Dr. and Mrs. Kumar, and my sister, Janavi Kumar, have been with me through all the joyful and troubled times I have experienced. Their unconditional love and advice has taken me this far and supported me in times of need.

During my first year as a Masters' student in Atlanta, the students in the Systems Realization laboratory helped make it a better learning environment. A special thanks to Patrick Chang, Ryan Stewart, Julie Bankston, Mukul Singhee, Jamal Wilson, Sarah Engelbrecht and Jane Chu for making the first year more memorable.

I would also like to thank the faculty and students in Georgia Institute of Technology, Savannah for helping in the difficult transition. Thanks to Andrew Hyder, Roger Jiao and Daniel Wu for their support last year.

I’d also like to thank Dr. Kenneth Gall and Dr. Suresh K. Sitaram for their support and guidance during the classes that helped provide me useful knowledge that I could use for the thesis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Page

NOMENCLATURE

P_c = Proportionality constant.

Tol_i = Tolerance of part i

Tol_{asm} = Assembly function requirement

d_i = Length of part i

W_i = Weight if part i

U_i = Fuzzy set for fuzzy factor i

A_i = Weight Vector for fuzzy factor i

μ = Degree of Membership

R_i = 1st order FCE Matrix for factor i

B_i = 1st order Fuzzy Comprehensive Set for factor i

I = Vector of weighted importance of fuzzy factors.

ς_i = Machinability of part i

ξ_i = Assembly sensitivity coefficient of part i.

ψ_i = Comprehensive factor of part i.

C_0 = Initial setup costs.

C_M = Total Machining Cost

l_i = Lower bound of Tolerance i

u_i = Upper bound of Tolerance i

l = Lower bound of Assembly Tolerance

u = Upper bound of Assembly Tolerance

b_0 = Intercept term in regression

b_i = Part-worth utility of level i in regression

x_i = Design attribute level i in binary matrix of regression

e = Error term in regression in regression

C_L = Costs due to quality loss

k= Taguchi Loss constant

m = Target performance value

RMS_e = RMS error in Cross Validation Procedure

SUMMARY

Tolerances play an important role in product fabrication. Tolerances impact the needs of the designer and the manufacturer. Engineering designers are concerned with the impact of tolerances on the variation of the output, while manufacturers are more concerned with the cost of fitting the parts. Traditional tolerance control methods do not take into account both these needs.

In this thesis, the author proposes a framework that overcomes the drawbacks of the traditional tolerance control methods, and reduces subjectivity via fuzzy set theory and decision support systems (DSS). Those factors that affect the manufacturing cost (geometry, material etc) of a part are fuzzy (i.e. subjective) in nature with no numerical measure. Fuzzy comprehensive evaluation (FCE) is utilized in this thesis as a method of quantifying the fuzzy (i.e. subjective) factors.

In the FCE process, the weighted importance of each factor affects the manufacturing cost of the part. There is no systematic method of calculating the importance weights. This brings about a need for decision support in the evaluation of the weighted importance of each factor. The combination of FCE and DSS, in the form of Conjoint Analysis (CA), is used to reduce subjectivity in calculation of machining cost. Taguchi's quality loss function is considered in this framework to reduce the variation in the output. The application of the framework is demonstrated with three practical engineering applications.

Tolerances are allocated for three assemblies; a friction clutch, an accumulator O-ring seal and a Power Generating Shock Absorber (PGSA) using the proposed framework. The output performances of the PGSA and the clutch are affected by the allocated tolerances.

On using the proposed framework, there is seen to be a reduction in variation of output performance for the clutch and the PGSA. The use of CA is also validated by checking efficiency of final tolerance calculation with and without use of CA.

CHAPTER 1. INTRODUCTION

1.1 Motivation

Globalization has interconnected markets and increased demand of goods to a variety of people and industries. This has led to an increase in competition within each industry, especially in mass consumption industries. For example, the sale of automotives in the United States of America is geared towards mass consumption. As a compromise, there are a lot of defects in the manufacturing processes. The 2009 JD Power and Associates Initial Quality Study (IQS) reports number of problems for each automaker within ninety days of ownership. According to the study, there is an average of 108 problems per 100 new vehicles sold [1]. This is a high number of defects to be encountered within the first ninety days of ownership. Also, the 2009 Consumer Reports [2], in Figure 1.1 b, revealed that 37% of all European, 41% of American brands and 6% of Asians car models have below average reliability ratings. Higher initial quality is required to reduce costs due to re-engineering and customer complaints. A higher quality rating also enhances an automakers' reputation for reliability [1]. Tolerances influence the final quality of the product.

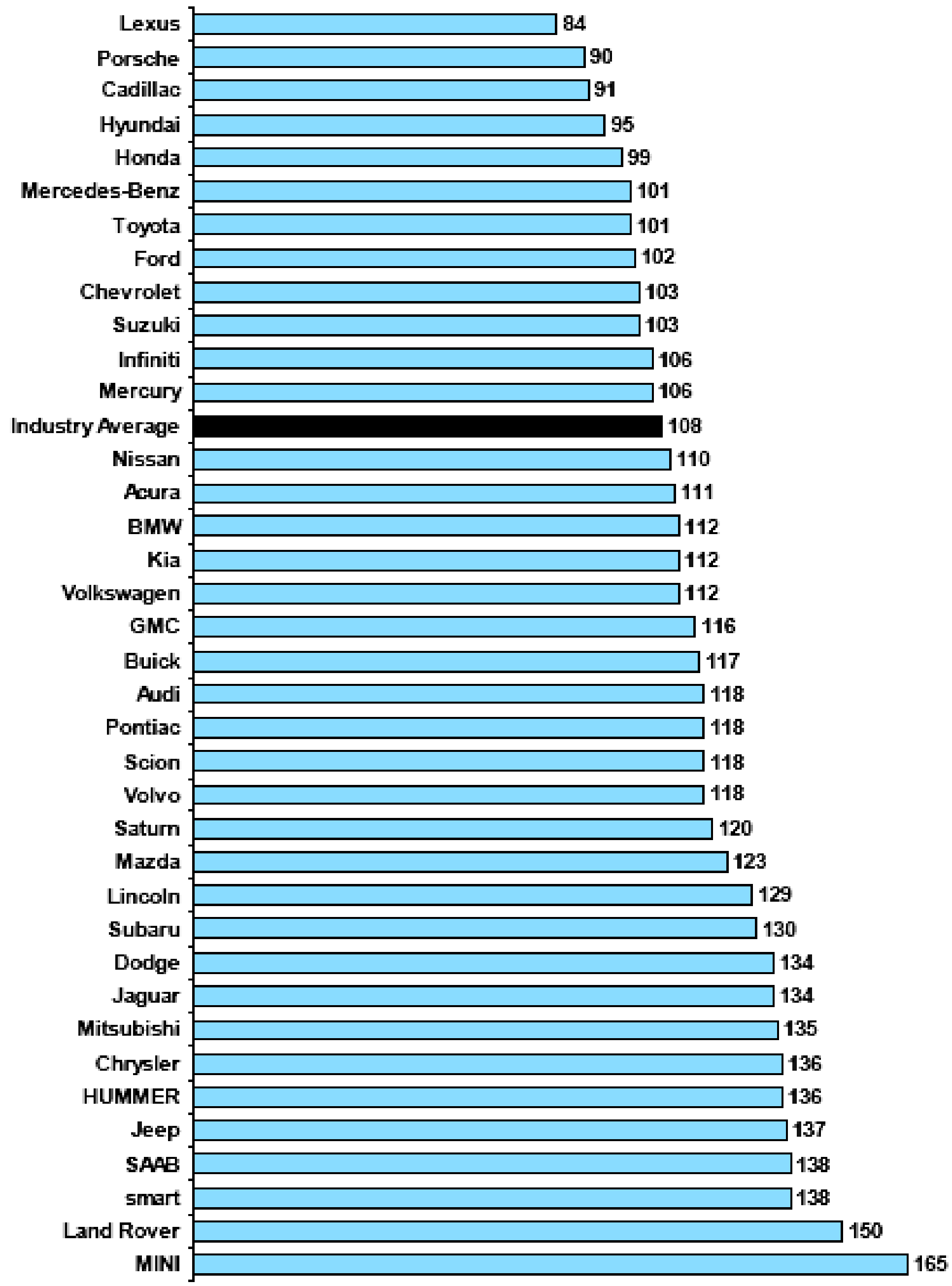


Figure 1.1 a:Problems per 100 Vehicles for 2009 J. D. Power and Associates Initial Quality Study [1]

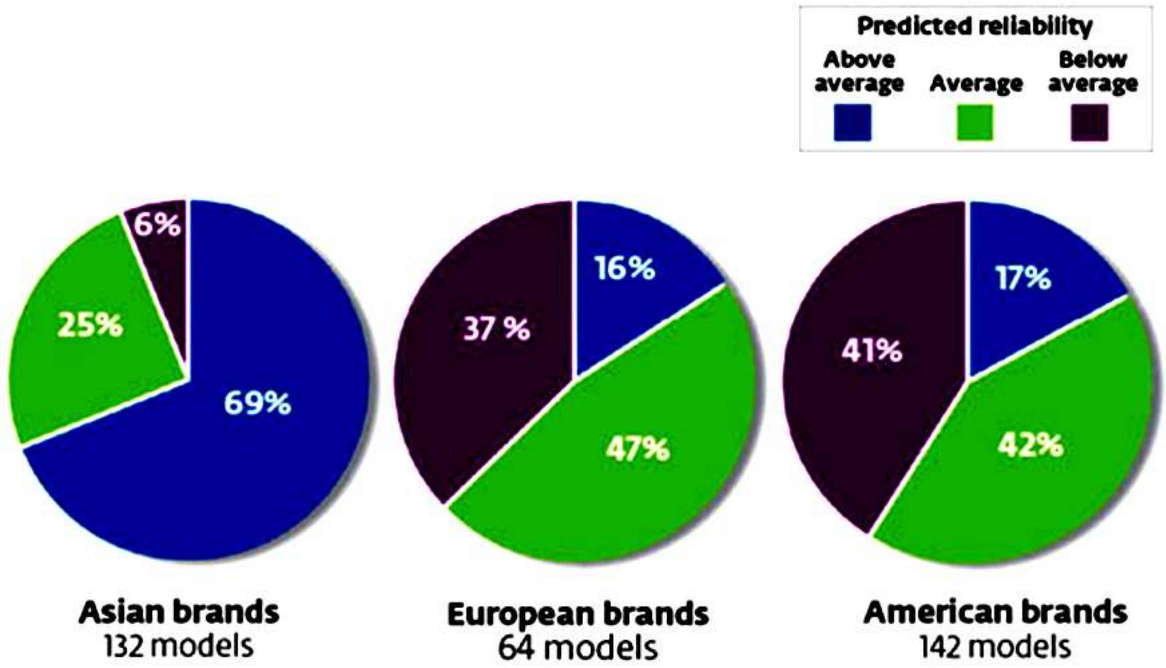


Figure 1.1 b: Consumer Reports 2009 Reliability Ratings [2]

1.2 Significance of tolerances

Tolerance is the variation in the dimension value of a part. It's not possible to manufacture a part with perfect dimension value, despite the amount invested in the manufacturing process. When many individual parts are assembled, there is an accumulation of tolerances [3]. Quantifying and controlling these tolerances can result in better system performance and reliability.

Tolerance requirements also dictate the selection of machining, tools and fixtures to be used for the product, operator skill levels, setup costs, inspection precision and gauging, scrap and rework. Every aspect of the product life cycle is affected, making it important to consider tolerances in the design of the product. It is important to allocate tolerances in a manner that reduces cost and does not jeopardize quality. Tighter tolerances increase the cost considerably, and may not suit the customer's needs, or could even have a negative impact on its life. Meager examination of component tolerances may result in the inability to create assemblies due to mismatched parts, or machines can perform in a limited manner.

Engineering designers and manufacturers are both concerned about the effect of tolerances. Engineers prefer tight tolerances to ensure a proper fit and lower variation in output performance. Manufacturers prefer loose tolerances, so that parts can be made easier and cheaper. Thus, specifications of tolerances form an important link between engineering design and manufacturing [4]. This relationship is summarized in Figure 1.2.

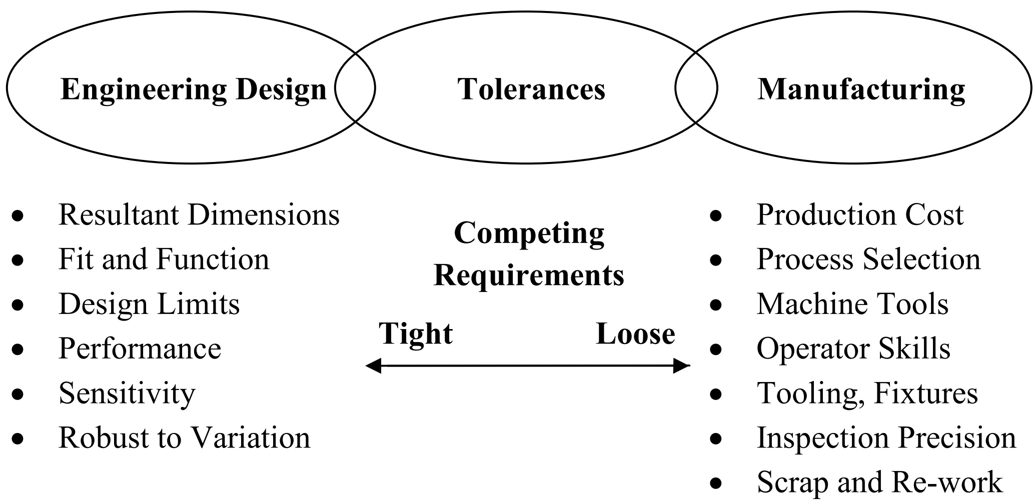


Figure 1.2: Tradeoffs between Engineering Designers and Manufacturers involved in assigning tolerances [4]

Over the past few decades, a majority of Fortune-500 companies have established comprehensive programs in quality management. These companies, some of whom are Motorola, IBM and Xerox, have programs improving tolerance specification, monitoring and control. Successful reduction in waste, reduction in cost and development time has led to an improvement in market share [4-5].

The main purpose of this thesis is to allocate tolerances using a method that incorporates the interests of the engineering designer and the manufacturers. The Fuzzy Comprehensive Evaluation (FCE) [6] method is considered to incorporate better estimation of machining costs. In the FCE, the machining costs are assumed to be dependent on certain 'fuzzy' variables (e.g. shape, material) that are subjective in nature and have no numerical measure. These factors are modeling using fuzzy sets [7], and a comprehensive evaluation is used to calculate machinability of each part. The machinability is directly proportional to the machining cost of the part. A part with higher machinability will have higher machining costs due to factors such as complex geometry, less malleable material etc.

In the FCE process, the weighted importance values for each of the factors affect the final machinability values. These importance weights were not assigned systematically. Thus, in the proposed research, the Conjoint Analysis (CA) method is introduced to provide a systematic method of deciding weights for each of the factors in the tolerance allocation procedure. Also, Taguchi's quality loss function is incorporated to reduce variation in output performance of the assembly.

1.3 Outline of thesis

The rest of this thesis is organized as followed: Previous methods in the field of tolerance allocation are summarized in Chapter 2. Their benefits and drawbacks are mentioned, leading to the use of Fuzzy Comprehensive Evaluation (FCE) to allocate tolerances. A detailed description of tolerance allocation using FCE is included in Chapter 3. The drawbacks are mentioned, which leads to the integration of Decision Support Systems and Taguchi Loss function into the FCE method.

In Chapter 4, Decision Support Systems, in the form of Conjoint Analysis (CA) is described in detail. To take into account the concerns of the designers, the Taguchi Loss Function is utilized to minimize variation in output performance. The quality improvement process, via utilization of Taguchi Loss Function, is described in Chapter 5.

In Chapter 6, the proposed method, which integrates CA and Taguchi's quality loss function into the FCE, is summarized. In Chapter 7, the proposed framework is applied to three different engineering problems. In Chapter 7.1, the method is used to allocate tolerances for a friction clutch assembly while reducing variation in clutch torque capacity. In Chapter 7.2, it does the same for an O-ring seal assembly in a hydraulic accumulator. In Chapter 7.3, the variations in output energy and vertical acceleration of a Power Generating Shock Absorber (PGSA) are minimized while allocating tolerances.

CHAPTER 2. TOLERANCE ALLOCATION

2.1 Tolerance Allocation and Tolerance Analysis

During tolerance specification, engineers need to determine whether to use tolerance allocation or tolerance analysis. The difference between the two concepts is illustrated in Figure 2.1. In tolerance analysis, the tolerance of each part is known and the final assembly tolerance is calculated. For tolerance allocation, the required final assembly tolerance is used to calculate the tolerance of each part in the assembly. The assembly tolerances in mechanical devices are generally due to accumulation of tolerances in assemblies of parts [8-9].

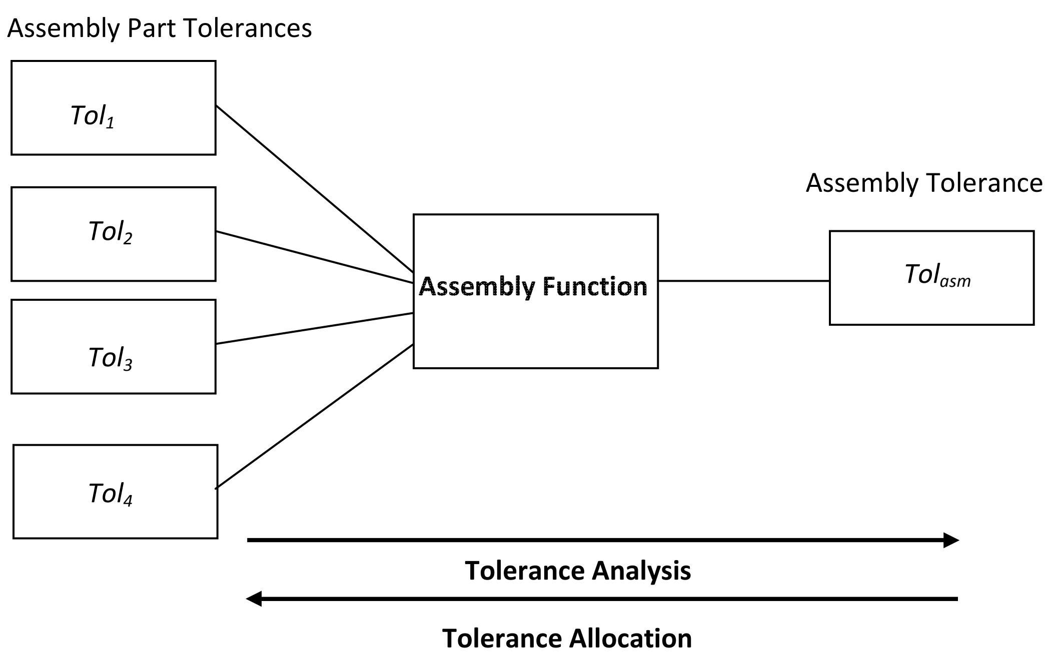
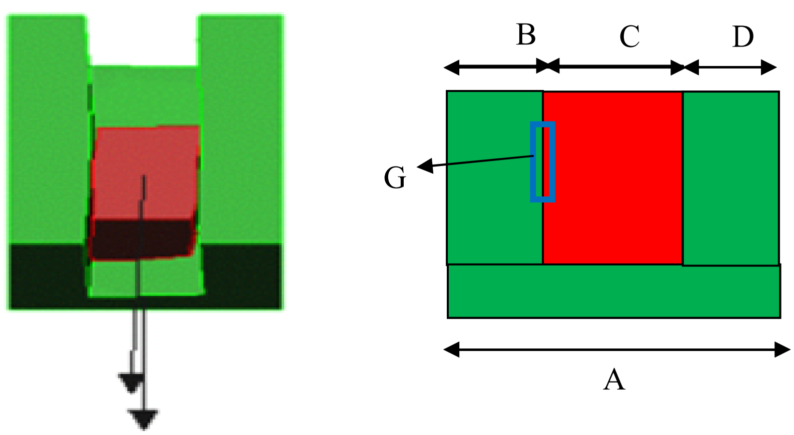


Figure 2.1: Difference between Tolerance Analysis and Tolerance Allocation

Tolerance Allocation is a method used to estimate the tolerance in the individual parts of assemblies given the final tolerance of the assembly (also called the assembly function requirement). It makes it possible for the designer to meet the clearance requirements in assemblies, while reducing manufacturing costs and improving efficiency. Tolerance allocation forms a common link for communication between the designer and manufacturer to help improve the overall production process [10].

An example of this process is provided by setting tolerances for parts in a block slider assembly. The figure used is shown in Figure 2.2 and the corresponding dimensions are displayed in Table 2.1. All dimensions are in inches. The gap/interference between B and C cannot be more than 0.01 inches. This is the assembly function requirement.



a) b)

Figure 2.2: Block-Slider assembly a) Isometric view b) Front view

The tolerance of the gap, Tol_G, is derived as follows. If all parts have zero tolerance and the gap G is required to be zero,

(2.1)

Once part tolerances are included in the equation:

$$(A \pm Tol_A) - (B \pm Tol_B) - (C \pm Tol_C) - (D \pm Tol_D) = Tol_G = Tol_{asm} \qquad (2.2)$$

Subtracting Equation (2.2) from Equation (2.1)

$$Tol_A - Tol_B - Tol_C - Tol_D = Tol_G = Tol_{asm} \qquad (2.3)$$

Table 2.1: Horizontal dimensions of assembly

Dimension	A	B	C	D
Length	4.5	1.6	1.3	1.6
Average Tolerances($\pm$)	0.03	0.004	0.005	0.006

2.2 Previous Methods of Tolerance Allocation

Several methods of tolerance allocation have been proposed in the past.

2.2.1 Proportional Scaling Method

A commonly used method, known as the Proportional Scaling Method (PSM), assigns tolerances based on process guidelines. They are then summed to check if they meet the assembly requirement. If not, they are scaled by a constant proportionality factor [8].

The value of the assembly function requirement is:

$$Tol_{asm} = Tol_A - Tol_B - Tol_C - Tol_D = 0.015 \qquad (2.4)$$

where Tol_i is the tolerance of part i and Tol_{asm} is the tolerance of the assembly. Tol_{asm} exceeds the maximum tolerance specification of 0.01 inches. To reduce this, the part tolerances are reduced by a proportionality factor P_C, as described by:

$$Tol_{asm} = P_C(Tol_A - Tol_B - Tol_C - Tol_D) \quad (2.5)$$

P_C is calculated to be 0.6667. After using the proportionality factor, the new tolerances are:

$$Tol_A = 0.6667(0.03) = 0.0201$$

$$Tol_B = 0.6667(0.004) = 0.0027$$

$$Tol_C = 0.6667(0.005) = 0.0033$$

$$Tol_D = 0.6667(0.006) = 0.004$$

This ensures that the maximum possible T_{asm} value is 0.01 inches, which meets the assembly requirements. This method can only implemented during the initial stages of tolerance allocation. The method requires prior knowledge of tolerances for it to be implemented. This is not practical when tolerance analysis is done during the design stage when natural tolerances of parts are unknown. Also, none of the factors that influence manufacturing cost are considered in this method.

2.2.2 Constant Precision Factor Method

Another approach is the Constant Precision Factor method, which allocates tolerances on the basis that the tolerances of parts are equal only if they are the same in size [11]. The size is defined as the cube root of its length [12]. The engineer does not need prior knowledge of the natural tolerances of the individual parts of the assembly, making it useful in designing new parts with unknown natural tolerances. The constant precision factor P_c is calculated by:

$$P_c = \frac{Tol_{asm}}{\sum(d_i)^{1/3}} \tag{2.6}$$

where tol_i is the tolerance of part i, d_i is the dimension of the part Tol_{asm}<0.01. The individual part tolerances are then calculated as:

$$Tol_i = P_c . d_i^{1/3} \tag{2.7}$$

To illustrate the procedure, the same block and slider assembly described in Figure 2.2, is used. The tolerances for A, B, C and D are described by its precision factor, as seen in Equation 2.8. The overall tolerance equation is:

$$Tol_{asm} = P_C(d_A^{\frac{1}{3}} - d_B^{\frac{1}{3}} - d_C^{\frac{1}{3}} - d_D^{\frac{1}{3}}) \tag{2.8}$$

The values for tolerances and dimensions are obtained from Table 2.1. The precision factor P_c is calculated to be 0.00562.

$$Tol_A = 0.00562(4.5)^{1/3} = 0.0093$$

$$Tol_B = 0.00562(1.6)^{1/3} = 0.00657$$

$$Tol_C = 0.00562(1.3)^{1/3} = 0.00613$$

$$Tol_D = 0.00562(1.5)^{1/3} = 0.00643$$

This method does not require prior knowledge of tolerance values, which is a major problem associated with proportional scaling method. However, it is also only used in the initial stages of tolerance allocation. It does not take into account factors such as shape, material etc. that also affect machining costs.

2.2.3 Allocation by Weight Factors

In the method of allocation by weight factors, weight factors are assigned to the tolerance of each part. A fraction of the tolerance is distributed to each part in the pool, depending on the weight factor of that part. A higher weight is assigned to those parts that are more expensive to manufacture or difficult to handle. This causes allocation of higher factors to those tolerances that are more costly, improving the performance of the design[8]. The same block and slider assembly described by Figure 2.2 and Table 2.1 is used to illustrate this process.

The tolerances are used to check if the assembly function requirement is below 0.01 inches, as shown in Equation 2.3. Since T_{asm} exceeds the tolerance limit, the weight factors are assigned. The tolerances for parts A, B, C and D are assigned weight factors of 30, 5, 5 and 20 respectively. The weights are assigned based on difficulty of machining the part. A part that is harder to machine is assigned a higher weight. The new tolerances for each part are given by:

$$Tol_i' = P_C W_i Tol_i \tag{2.9}$$

where P_C is the proportionality constant, W_i is the weight factor of part i, T_i is the tolerance of part i and $\sum W_i$=1. The tolerance equation becomes:

$$Tol_{asm} = P_C[\left(\frac{10}{60}\right)Tol_A - \left(\frac{20}{60}\right)Tol_B - \left(\frac{10}{60}\right)Tol_C - \left(\frac{20}{60}\right)Tol_D] \tag{2.10}$$

On solving the equation using tolerance values for Table 2.1 such that the final Tol_{asm} value is 0.01, a P_c value of 0.8163 is obtained. The tolerance values for A, B, C and D are re-allocated as:

$$Tol_A = 0.8163\left(\frac{30}{60}\right)(0.03) = 0.0122$$

$$Tol_B = 0.8163\left(\frac{5}{60}\right)(0.004) = 0.00027$$

$$Tol_C = 0.8163\left(\frac{5}{60}\right)(0.005) = 0.00034$$

$$Tol_D = 0.8163\left(\frac{20}{60}\right)(0.006) = 0.0016$$

The allocation by weight factors considers the machining difficulty important in allocation tolerances. However, there is no systematic way of assigning weight factors for the tolerance of each part. It is hard to assign weight factors for each part accurately, unless there is prior machinability data available for each part. This method also requires suffers the same disadvantage as the proportionality factor method, as it requires prior knowledge of the natural tolerances of each part.

2.2.4 Allocation Using Least Cost Optimization

A more promising method of tolerance allocation involves evaluating machining costs of each component. The relationship between the machining costs and part tolerance is expressed through a mathematical formula, and the total machining cost is optimized to a minimum. It is subject to the constraints of the assembly function requirements.

To achieve this, there is a need for cost-tolerance data for each part in the assembly. A lot of models based on the cost tolerance relationship have been proposed in the past [13]. A few of these are listed in Table 2.2.

Table 2.2: Cost-Tolerance Models

Cost Model	**Equation**	**Reference**
Linear	$X-Y(Tol_i)$	[14]
Exponential	$Ye^{-k(Toli)}$	[15-17]
Combined Linear and Exponential Model	$X+Y(Tol_i)+Ze^{-(Toli)}$	[18-19]
Cubic Polynomial	$X+Y(Tol_i)+Z(Tol_i)^2+M(Tol_i)^3$	[19]
Fourth Order Polynomial	$X+Y(Tol_i)+Z(Tol_i)^2+M(Tol_i)^3+N(Tol_i)^4$	[19]
Reciprocal	$X+Y/(Tol_i)$	[20-21]
Reciprocal Squared	$X+Y/(Tol_i)^2$	[22]
Reciprocal Power	$X+Y/(Tol_i)^k$	[23]
Multi/Reciprocal Powers	$Y/(Tol_i)^k$	[9, 24-26]
Modified exponential Model	$Ye^{-k(Toli-m)}$; $Tol_{min}<Tol_i<Tol_{max}$	[27]
Exponential/ Reciprocal Power Hybrid	$Ye^{-m(Toli)}/Tol_i^k$	[28]
Discrete Model	Discrete points	[29]

where X, Y, Z, M, N, k, m are all constants. X represents the fixed costs which include setup cost, tooling, material, prior operations etc. The Y term is the cost of manufacturing of the specified component that is related to the tolerance of part Tol_i. The exponent k represents the sensitivity of the cost to changes in tolerance values. The cost is calculated for each part and then summed together to obtain total cost. The total cost is optimized to a minimum considering the tolerances as design variables.

There has been very little verification for each of these curves. Manufacturing cost data are very dependent on location, materials, tooling, overheads etc. For this reason, manufacturing cost data are not published [8].

2.3 Drawbacks of Existing Methods

Based on the methods described above, a list of the drawbacks of the existing methods is listed in Table 2.3.

Table 2.3: Drawbacks of Existing Methods

Method	Description	Drawbacks
Proportional Scaling Method	Tolerances scaled by a proportional factor to meet assembly requirements	The method requires initial knowledge of tolerances
Constant Precision Factor Method	Tolerances allocated proportional to cube root of dimension size	The method does not take into account factors that affect machining costs (such as shape, material etc.)
Allocation by Weight Factors	Weights are assigned to the tolerance of each part depending on machining criteria	There is no systematic method of assigning weights.
Least Cost Optimization	Assumes cost tolerance data relationship	The initial cost-tolerance data hard to obtain for every part

Based on these drawbacks, the first research question is formulated.

2.4 Research Question and Hypothesis

Is there a systematic way to allocate tolerances taking into account the factors that affect Machining Cost (e.g. dimension, shape, material) when there is no initial knowledge of tolerances?

In this thesis, the Fuzzy Comprehensive Evaluation (FCE) [6] method is considered to incorporate better estimation of machining costs. In the FCE, the machining costs are assumed to be dependent on certain ‘fuzzy’ variables (e.g. shape, material) that are subjective in nature and have no numerical measure. These factors are modeling using fuzzy sets [7], and the FCE is used to calculate machinability of each part. Machinability is a measure of the machining difficulty of a part. A part with higher machinability will be more expensive to machine and will have looser tolerances. This method is described in detail in Chapter 3.

CHAPTER 3. TOLERANCE ALLOCATION USING FUZZY COMPREHENSIVE EVALUATION PROCEDURE

3.1 Fuzzy Set Theory

A majority of engineering courses do not sufficiently address the uncertainty that exists in current engineering models. In real life multidisciplinary design, uncertainty exists as a significant part of all abstractions, models and solutions. In such an environment, it is important to obtain precise solutions that are insensitive to small variations in the model's parameters and variables. Achieving increasing levels of precision requires increases in cost and time. The more complex a system is the more uncertainty there is in that system. Real world systems have a large deal of complexity, meaning that traditional methods of analysis are too precise to be implemented. It is thus important to balance the degree of precision with the associated uncertainty [7].

Many researchers assume that the uncertainty in the parameters is due to randomness and these can be determined by estimating the probability distribution of their variation. Stochastic programming and probability theory are used to obtain the solution. Once probabilistic constraints are obtained, techniques such as Monte Carlo simulations and Latin Hypercube sampling are used to obtain the probability of failure of each constraint. However, there are areas where it is not possible to obtain accurate statistical information. It is not possible to use the probabilistic method for these applications, since the improper modeling of uncertainty would cause a greater error in the solution [30].

Possibility-based design (or fuzzy set theory) methods have been recently used in problems with insufficient statistical information. The fuzzy analysis preserves the randomness of the variables, and allows for more conservative designs. The main advantage of this method is that it is easier to define the fuzzy variable than the random variable when there is limited statistical data available [30]. Also, fuzzy operations are simpler to use than statistical operations.

3.2 Evaluation of Part Machinability Using Fuzzy Comprehensive Evaluation

Before allocating the tolerances, it is necessary to evaluate the machinability of each part. The machinability refers to the difficulty involved in machining a part. A higher value of machinability indicates a higher machining cost for the part. This implies that looser tolerances will be allocated for the part.

In accordance with design and machining criteria, certain factors are significant in the evaluation of machinability of parts. These factors are 'fuzzy', since they are subjective in nature with no numerical measure. They are modeled using fuzzy sets [7]. In the Fuzzy Comprehensive Evaluation (FCE) [31] process, the machinability value is calculated based on the fuzzy factors. Typically, the following factors, which most influence machinability, are considered in the FCE:

Dimension Size (DS): The Dimension Size of an assembly component refers to its characteristic length, which describes its' overall size.

Geometric Structure (GS): The Geometric Structure of a part is a relative value that defines total cost of the end product. It is a measure of the machining difficulty caused by the shape of the assembly part. Symmetric shapes usually have lower GS values than irregular shapes.

Material Machinability (MM): Material Machinability of an assembly component refers to its parent material malleability. A material with higher malleability is easier to machine.

Process Accuracy (PA): The process accuracy for a component is the required precision that a component needs to be machined with. A component that is used more in an assembly will have a higher process accuracy than one that is not used.

Housing Retainer

Figure 3.1: Housing and Retainer of a Ball Bearing Assembly

Figure 3.1 displays a ball bearing's housing and retainer. The effect of the four fuzzy factors on machinability values are compared for each part. If only the factor DS is used to evaluate machinability, the housing has higher machinability if tolerances are allocated in the Y direction. If only GS is considered, the retainer has higher machinability because it has a more complex geometry that is harder to machine, while the factor MM depends on the materials used. A material with higher malleability will be harder to machine resulting in higher malleability. Using only PA values, the retainer will have higher machinability since it is used more frequently in the functioning of the ball bearing assembly. The typical process of the FCE for each component of the assembly is summarized in Figure 3.2.

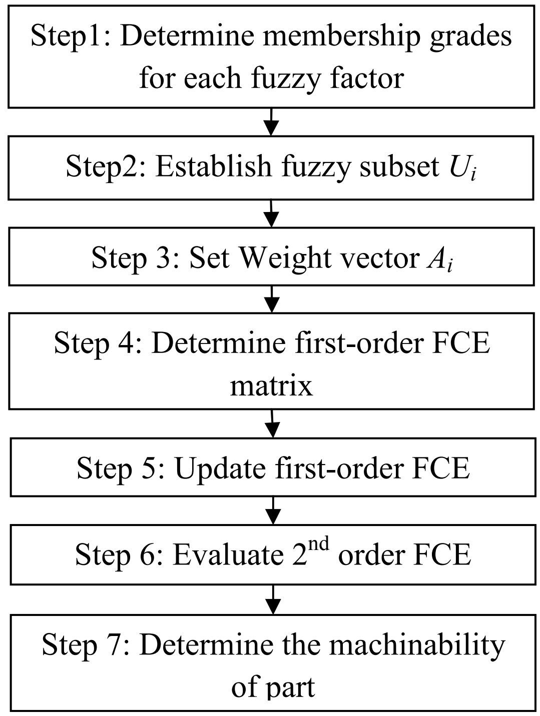


Figure 3.2: Fuzzy Comprehensive Evaluation of Each Assembly Part

Step 1: Elicit Degrees of Membership for Each Fuzzy Factor

The fuzzy factor set is developed for each fuzzy factor through use of pairwise comparisons [7]. Each of the four factors (DS, GS, MM, and PA) is divided into different grades that enable quantifying the fuzzy factors. The preferences of a group of experts are used to assign membership degrees to the fuzzy variables. Ranking is done through comparison of pairs of fuzzy grades, and this determines membership degree values. The fuzzy grades are empirical and are modified for each application. An example of this is demonstrated with the elicitation of membership degrees for Material Machinability (MM) of Aluminum among 100 field experts in the following example:

Example: There are 100 experts who are used to indicate the Material Machinability (MM) value of an aluminum cylinder. There are three grades for MM; Poor, Medium and Good. Table 3.1 summarizes the survey. Out of 100 people, 75 people preferred to use the term 'Medium' over 'Poor' to describe the MM of aluminum; 100 people preferred to use 'Good' over 'Poor'; 80 preferred to use the term 'Good' over 'Medium'. This is an anti-symmetric matrix that follows a reciprocal relationship. The total number of responses is 300. The percentage preferences are used to obtain degrees of membership of each grade and shown in Figure 3.3.

Table 3.1: Membership Degree Elicitation using Preferences Matrix

	Number who preferred		
	Poor	Medium	Good
Poor	-	75	100
Medium	25	-	80
Good	0	20	-
Total	25	95	180
Percentage	8.33	31.67	60

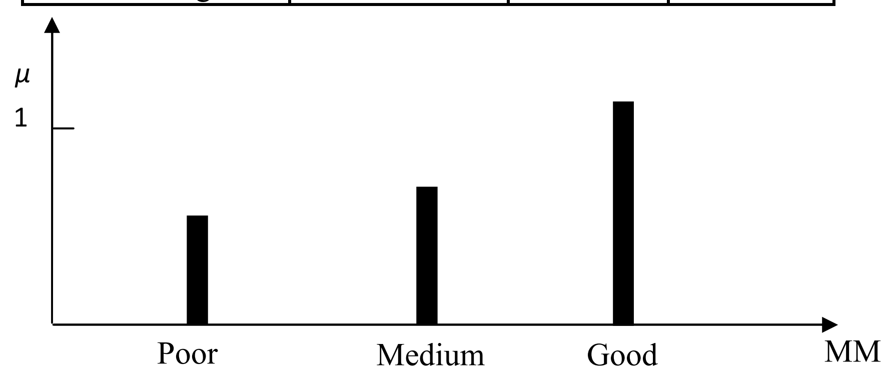


Figure 3.3: Membership Degrees (μ) for MM of Aluminum Cylinder

Step 2: Select the Fuzzy Subset U_i

This is evaluated from the membership degree values for each factor. The membership values of the factor in each grade is the fuzzy subset U_i are defined as,

$$U_i = (u_{i1}, u_{i2}, u_{i3} \dots \dots . u_{in}) \tag{3.1}$$

where n is the number of grades and u_{ij} denotes the membership value of j^{th} grade for the i^{th} factor. The fuzzy definitions for the four factors depend on the application.

Step 3: Set Weight Vector (A_i) for Fuzzy Factor

The weight vector set i^{th} factor, A_i, can be derived from the fuzzy subset U_i

$$A_i = (a_{i1}, a_{i2}, a_{i3} \dots \dots . a_{in}) \tag{3.2}$$

where $a_{ij}=u_{ij}/\sum u_{ij}(j=1,2,\dots\dots..n)$, u_{ij} denotes the membership value of j^{th} fuzzy grade for the ith factor. This step ensures that the fuzzy factor is divided between membership values of 0 and 1.

In the FCE, the level of machinability is between 0 and 1. It is divided into ten equally spaced levels.

$$\tau = \{A, B, C, D, E, F, G, H, I, J\} \tag{3.3}$$

Thus, it is determined that τ can be $\{0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1\}$. The lowest level is assumed to be the easiest to manufacture. The highest level represents the level in which the part is most difficult to manufacture. Machinability level A (value 0.1) is the level which creates a part that is easiest/cheapest to machine. Machinability level J (value 1) is the level which creates the part that is hardest/most expensive to machine.

Step 4: Determine 1st order FCE Matrix

The first order FCE matrix can be determined based on the experience of experts.

The matrix R is determined for each fuzzy factor. In the matrix, the membership degrees of every grade in ten machinability levels are determined. A panel set of industry experts are asked to vote for the most appropriate machinability level for each grade. Based on voting percentages obtained from experts, the FCE matrix can be constructed as:

$$R = \begin{bmatrix} k_{11} & k_{12} & . & k_{1p} \\ k_{21} & k_{22} & . & k_{2p} \\ . & . & . & . \\ . & . & . & . \\ k_{n1} & k_{n2} & . & k_{np} \end{bmatrix} \quad (3.4)$$

where k_{ij} is the membership of fuzzy grade i $(i=1,2\ldots\ldots n)$, in machinability level j (j=1……10). The values are empirical and are modified for specific applications.

Step 5: First Order Fuzzy Comprehensive Set

The first order Fuzzy comprehensive set for every factor i, B_i, can be obtained using

$$B_i = A_i o R_i = (a_{i1}, a_{i2}, \ldots\ldots a_{n1}) o \begin{bmatrix} k_{11} & k_{12} & . & k_{1p} \\ k_{21} & k_{22} & . & k_{2p} \\ . & . & . & . \\ . & . & . & . \\ k_{n1} & k_{n2} & . & k_{np} \end{bmatrix}$$

$$= (B_{i1}, B_{i2}, \ldots\ldots B_{ip}) \quad (3.5)$$

Where p=10. Once the evaluations, in Equation (3.4), are done for each factor, the first-order FCE matrix, R_{new}, can be obtained by combining the calculated B_i matrices for all the factors,

$$R_{new} = \begin{bmatrix} B_1 \\ B_2 \\ . \\ . \\ B_n \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & . & B_{1p} \\ B_{21} & B_{22} & . & B_{2p} \\ . & . & . & . \\ . & . & . & . \\ B_{n1} & B_{n2} & . & B_{np} \end{bmatrix} \tag{3.6}$$

where p=10. This calculates the membership degree of each fuzzy factor in ten machinability levels.

Step 6: Evaluate Second Order FCE Matrix

The final step of the fuzzy comprehensive evaluation is to calculate the fuzzy set B for the ten different levels;

$$B = I * R_{new} = (b_1, b_2, \dots, b_p) \tag{3.7}$$

where p=10. I denotes the weighted importance of each factor. For instance I= $(i_{DS}, i_{GS}, i_{MM}, i_{PA})$. This calculates the machinability value in each of the ten levels.

Step 7: Determine Machinability of Part

The machinability is generally evaluated using the weighted average method. The weighted average of the fuzzy machinability levels can be evaluated by

$$\zeta = \sum_{p=1}^{10} B_p \tau_p \Big/ \sum_{k=1}^{10} B_k \tag{3.8}$$

where τ_p is the machinability at level p, b_k is the second order FCE matrix. This is the final machinability of the part.

3.3 Allocation by Determining Cost Function

Once the machinability values are calculated for each component using the FCE and CA, the tolerance allocation is conducted. The relationship between the assemblies dimensions are assumed to be described by:

$$D_0 = f(D_1, D_2 \ldots\ldots D_k) \tag{3.9}$$

where D_0 is the required assembly function requirement, D_i is the i^{th} dimension variable. When tolerances are added to each dimension, Equation (3.9) changes to:

$$D_0 + \Delta D_0 = f((D_1 + \Delta D_1), (D_2 + \Delta D_2) \ldots\ldots (D_k + \Delta D_k)) \tag{3.10}$$

After applying Taylor's expansion and omitting the higher terms, Equation (3.10) becomes

$$D_0 + \Delta D_0 = f(D_1, (D_2 \ldots D_n) + \frac{\partial D_0}{\partial D_1}\Delta D_1 + \frac{\partial D_0}{\partial D_2}\Delta D_2 \ldots\ldots + \frac{\partial D_0}{\partial D_n}\Delta D_n \tag{3.11}$$

$$\Delta D_0 = \frac{\partial D_0}{\partial D_1}\Delta D_1 + \frac{\partial D_0}{\partial D_2}\Delta D_2 \ldots\ldots + \frac{\partial D_0}{\partial D_n}\Delta D_n \tag{3.12}$$

$$\Delta D_0 = \sum_{i=1}^{n} \frac{\partial D_0}{\partial D_i}\Delta D_i = \sum_{i=1}^{n} \xi_i \Delta D_i \tag{3.13}$$

where ξ_i is the degree of importance of each part tolerance on the assembly tolerance. It is also known as the assembly sensitivity coefficient. Assume $Tol_i = \Delta D_i$ and $Tol_{asm} = \Delta D_0$. Equation (3.13) now becomes:

$$Tol_{asm} = \sum_{i=1}^{n} \frac{\partial f}{\partial D_i} Tol_i = \sum_{i=1}^{n} \xi_i Tol_i \tag{3.14}$$

This is the assembly function equation. The degree of importance value, ξ_i, controls the value of tolerance allocation. It emphasizes the degree of importance of the tolerance of each component in an assembly. The larger the value of g_i is, the lower the corresponding tolerance is. The comprehensive factor, ψ_i, for part *i*, is calculated

$$\psi_i = \frac{\zeta_i}{\xi_i^2} \tag{3.15}$$

where ζ_i is the machinability for part *i*.

The final model of the tolerance allocation is obtained using the reciprocal model. The optimal tolerance allocation model is described by:

$$C_M = C_0 + \sum_{i=1}^{n} \frac{\psi_i}{tol_i} \tag{3.16}$$

subject to $l_i < tol_i < u_i$, $1 < i < n$, $l < tol_{asm} < u$

where C_M is the total machining cost, C_0 is the setup costs, which is constant constant, $L=\{l_1, l_2, \ldots l\}$ and $U=\{u_1, u_2 \ldots . u_n\}$ are the constraint vectors for the upper and lower tolerance limits of assembly components. It is required to minimize the cost *C*, through optimization. This is done by utilizing any standard algorithm of the optimization methods will be applicable.

3.4 Significance of Weighted Importance Vector *I*

In Section 3.3 Step7, different weighted importance values for the fuzzy factors result in different values of machinability. Consider the same example of two parts of the ball bearing assembly in Figure 3.1. Only two fuzzy factors (DS and GS) are used to determine machinability. If the DS has a much higher weighted importance and tolerances are allocated in the vertical direction, the housing has a higher machinability value. If the GS has a much higher weighted importance value, then the retainer has higher machinability since it has a more complex shape to machine.

3.5 Research Questions and Hypothesis

The above details describe the basis for the following research questions in this thesis.

1) *The method FCE requires calculation of weighted importance values for each attribute. These are normally assigned by asking experts to rate each attribute for their worth. All the four attributes are chosen because they are important. So it will be difficult to accurately decide on the weighted importance. Is there a better and more systematic method to achieve this purpose?*

Conjoint Analysis (CA), a method used often in marketing field, is seen as a method to solve this problem. Conjoint Analysis is a systematic method for creating and ranking a set of many design configurations based on design attributes to model designer preferences. It is a study of trade-offs. It is based on two concepts: Attributes, e.g. Foods and price; and Levels, e.g. Spinach, pizza, rice (food) and \$40, \$10, \$5(price). A combination of attribute levels e.g. \$40 pizza, is called a product concept. In a conjoint analysis, consumers are asked to rate product concepts instead of rating each individual attribute of a product. It is easier to answer the question “Do you prefer spending money on \$40 pizza as opposed to \$10 rice” instead of “How much more important is the attribute Food over Price”. Conjoint Analysis produces a set of utilities that measure accurately a consumer’s preferences for an attribute.

In the framework, the fuzzy factors are the attributes and the grades are the levels. Conjoint Analysis is used to determine the weighted importance of the fuzzy factors. The CA procedure is described in detail in Chapter 4.

2) *The Fuzzy Comprehensive Evaluation procedure includes factors that affect the maching costs, the costs incurred before a product is sold. However, there are also costs associated with the output quality once the product is produced. Any variation in the output quality parameter would reduce the worth of the assembly.*

To solve this problem, Taguchi's quality loss function is introduced. A performance measure (e.g. torque capacity for a clutch) is defined. The variation of the measure with the tolerance value is minimized during the optimization process. This reduces losses due to variation in output performance of the assembly. The use of Taguchi's quality loss function is described in detail in Chapter 5.

CHAPTER 4. DECISION SUPPORT PROCESSES

4.1 What are Decision Support Systems?

Decision Support Systems(DSS) are interactive computer-based systems that assist the decision maker (DM) use available data and models to make decisions[32]. They are generally utilized by managers to assist in semi-structured or unstructured decision making processes [33].

DSS evolved from two main areas of research, one at the Carnegie Mellon Institute of Technology in the late 50's and early 60's (Simon, Cyert, March and others). The other area was the technical work carried out at Massachusetts Institute of Technology in the 1960's [34]. Classic DSS tool design is used for components that provide different types of services. Some of these include:

a) Database management components which are capable of dealing with data, information and knowledge.
b) Computationally powerful modeling functions that are managed by model management system.
c) Simple, powerful GUI(Graphical User Interface) Designs [35].

Since DSS were first developed, it has evolved to help support decision making in specific problems.

There have been many attempts to improve the efficiency and effectiveness of decision making through use of DSS [36]. The development of information technology in the period of computer industry growth, which saw high yields in data processing (DP), microcomputers and networks helped [37]. DSS in UNIX systems were prominent in the late 1970's and moved to Windows in the early 1990's. During the 1990's, rapid spread of information to DM's through the internet has led to an increase in applications for DSS. High efficiency in decision making is another byproduct of the internet area. This is because the web browser serves as a user interface that is easy to understand and utilize. In the last five years, the use of mobile phones to access electronic services has increased rapidly. This has expanded the accessibility of tools to decision makers who are not located at their desktops [35].

In the case of the Fuzzy Comprehensive Evaluation method described in Chapter 3, there is a need for decision support in the evaluation of weighted importance of fuzzy factors.

4.2 Conjoint Analysis

Conjoint Analysis (CA) is a method often used in the marketing field for determining a quantitative value for a decision maker's preferences during the evaluation of a multi-attribute problem. CA is beneficial as a decision-making process since the tool is a systematic method for creating and ranking a set of many design configurations based on design attributes to model designer preferences. The attribute of each design objective is a measurable quantity that can be used to represent the value of each objective. This process creates a discrete number of configurations to rank. A multiple regression analysis of the ranking is done, which allows for an easy, systematic method for modeling preferences. A design selection method based on a rank ordering of all design alternatives is beneficial because it tells which is ‘‘best,’’ and gives insights as to the ordering of preference of the other alternatives [38].

CA is a method that has been used by marketers’ consistently for the last three decades to evaluate customer preferences [39]. There are three types of CA; Conjoint Value Analysis [40], Adaptive Conjoint Analysis [41] and Choice-based Conjoint Analysis [42].

Conjoint Value Analysis (CVA), also known as the traditional full profile CVA, is a simple evaluation of CA that can be implemented by hand. Computers can also be used to speed up the process. It can be used for problems that contain up to 6 attributes [43]. The main problem with this method is that there is an increase in possibility of error as the number of attributes increase. As the number of attributes increases, more combinations need to be ranked, which increases user fatigue.

Adaptive Conjoint Analysis (ACA) is an improved method that has been developed to handle more attributes. It utilizes a hybrid approach that combines state evaluations of attributes and levels with pair-wise comparisons. This causes a reduction in number of comparisons made. The interviewing process for implementing ACA adapts to the respondent’s answers as the survey progresses. The answers to the preceding questions are thus used to determine the subsequent questions, making this method harder to implement.

Choice-Based Conjoint (CBC) Analysis is a more involved implementation of CA that is similar to ranking of products in the competing market. Respondents choose their preferences for products from a set of possible alternatives instead of using rating or ranking scales. In addition, the respondent does not choose to purchase any of the products, as in the real world.

Conjoint Analysis can also be applied to engineering applications. To be specific, CVA is used due to its ease of implementation and ability to be used in engineering systems with less than six attributes [43]. The flow chart shown in Figure 4.1 represents the typical framework.

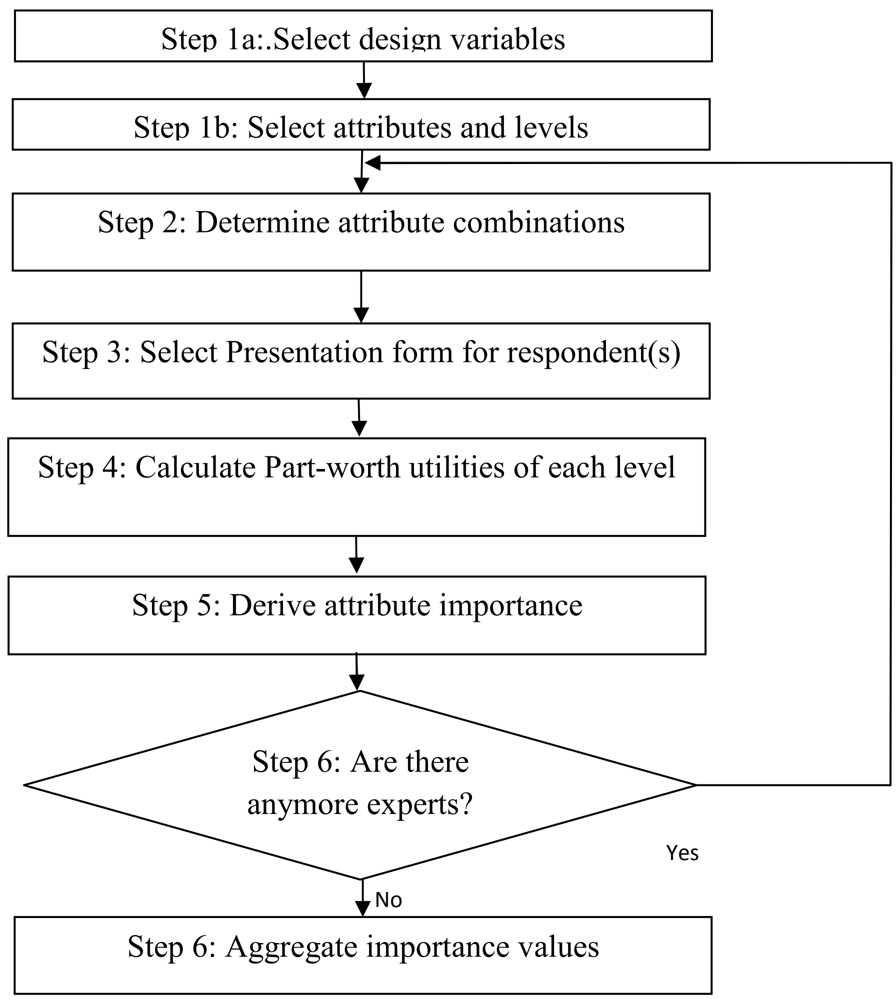


Figure 4.1: Flow Chart for Conjoint Analysis

Step 1: Determine Attributes and Levels

The first step is to determine the most important design attributes for the given problem. This can be done by any means from a simple design team discussion to an in-depth analysis of the problem involving customers, designers, etc. to see which design objectives are most important. These objectives must be a function of the necessary design variables. The attribute of each design objective is a measurable quantity that can be used to represent the value of each objective. For example, the attribute for cost of a product would be number of dollars spent for each unit.

Once the attributes are determined levels must be created for each. Choosing levels can be difficult for engineering applications as most of them involve continuous attribute values rather than known discrete values. The decision can be simplified if there are specific bounds on an attribute based on the specific design problem, previous expertise, or existing knowledge of the system.

Step 2: Determine Attribute Combinations

Once the levels are chosen, the next step is to create a number of design alternatives. The number of combinations has a direct impact on the complexity of the evaluation process and on the accuracy of the part-worth calculation. Factorial evaluations [44] can be used to determine the number of combinations. A full factorial design will give the most accurate evaluation as it uses every combination of each level possible. For problems such as this where the number of combinations in a full factorial are too large to rank, a fractional factorial can used to lower the number of alternatives. A fractional factorial [45] design will take an adequate fraction of combinations from the full factorial design with as little effect on the overall represented results as possible.

Step 3: Select Presentation Form/Nature of Judgment

After the combinations are made, the method of presentation of alternatives and the nature of the judgment is chosen. The most basic methods of presentation are verbal, paragraph, and pictorial description. Then, the presentation form for judging is selected (e.g. ranking or rating) to measure which alternatives are more favorable. This is where the DM's preferences are incorporated in the design. Due to the applicability of CA to gaining input from multiple DMs, this portion of the method could be done for one or many rankings or ratings. Depending on the number of DM's the results may need to be aggregated to get the preferences models representing the entire decision population. A running average can be a simple and accurate method for aggregation especially when mass customer surveys are involved.

Step 4: Calculate Part-Worth Utilities via Dummy-Variable Regression

The part-worth values for each level of each attribute represent the relationship between the objective attribute values and the corresponding DM's preferences. Regression techniques are common for the determination of these values and provide high accuracy. Dummy-Variable Regression technique [46], which is employed in this method, uses a binary matrix representation of each attribute combination to determine the part-worth values. A method known as Effects Coding[47] is also used to eliminate inconsistencies in the resulting part-worth values due to the possibility of the statistically significant intercept term. The rating data is fit to a regression model of the form,

$$y = b_0 + b_1 x_1 + b_2 x_2 + \cdots + b_n x_n + e \tag{4.1}$$

where y is the rating value, b_0 is an intercept term, $b_1, b_2, \ldots, b_n$ are the part worth utilities of the $x_1, x_2, \ldots, x_n$ design attribute levels and e is an error term. In the case where ranking is used to measure the customer's preference, a logit transformation of the given ranking value is required. More information on logit coding and effects coding is provided in Refs [46-48].

Step 5: Determine Attribute Importance.

The individual part-worth utilities of the levels for each factor are used to determine the attribute importance values for each factor. The relative importance of each attribute is the difference each attribute makes in the total product utility. The difference is the range of the part-worth utility values for all levels in that attribute. The ranges are directly proportional to the attribute importance and are used to calculate the percentage importance values of each attribute [49].

Step 6: Aggregate Importance Values

When aggregating importance attributes for multiple experts, it is best to average importance values obtained for each individual instead of obtaining importance values using the average utility. For example, a bunch of respondents are asked to choose between two brands, A and B. If half of the respondents preferred each band, the average utilities of A and B would be the same, implying that the importance of brand would be zero [49].

Attribute importance are scaled by ratio and relative. An attribute with an importance of 30% is three times as important as an attribute with an importance of 10%. It is always relative to the other attributes being used in the study. It is possible to compare importance of attributes' within a conjoint study but not across studies that feature different lists of attributes [49].

4.3 Application of Conjoint Analysis

A simple example of the entire process is shown. Consider the problem of a consumer buying a car. It is a problem with three attributes: Brand, Mileage, and Price. There are three available Models; Mitsubishi (M), Ford (F) and Toyota (T). The customer has a taste for two colors; Black (B) and Green (G). The customer is willing to spend $5000-$15000. Price is divided into three levels $5000, $10000, and $15000. The set of combinations is created using a full factorial design resulting in 18 total possible combinations. The 18 combinations are shown in Table 4.1.

$$3\ Models\ x\ 2\ Colors\ x\ 3\ Prices = 18\ Combinations \tag{4.2}$$

The chosen presentation form and nature of judgment is a comparison of all 18 combinations based on a rating scale from 1 to 10 (10 being the best) for simplicity. To further simplify the example, it is assumed that there is only one DM for this analysis.

Table 4.1: Full Factorial Design for Dummy-Variable Example

Combination	***Model***	***Color***	***Price***
1	Mitsubishi	Green	$5000
2	Mitsubishi	Green	$1000
3	Mitsubishi	Green	$1500
4	Mitsubishi	Black	$5000
5	Mitsubishi	Black	$1000
6	Mitsubishi	Black	$1500
7	Ford	Green	$5000
8	Ford	Green	$1000
9	Ford	Green	$1500
10	Ford	Black	$5000
11	Ford	Black	$1000
12	Ford	Black	$1500
13	Toyota	Green	$5000
14	Toyota	Green	$1000
15	Toyota	Green	$1500
16	Toyota	Black	$5000
17	Toyota	Black	$1000
18	Toyota	Black	$1500

The next step is to gain the respondent's preferences for each of the above combinations through customer surveys, computer programs, or elicitations from designers. As mentioned in the introduction to this chapter, the respondent can be a customer or user of the product or even the designer or DM conducting the CA. In either case the subjective data elicited are based on the attributes and preferences on the respondent. Therefore the suggested final design will represent the preferred design as pertains to the respondent(s) giving the rating/ranking data. The ratings for this example are presented in Table 4.2.

Table 4.2: Respondent Rating of Attribute Combinations

Combination	*Model*	*Color*	*Price*	*Ranking*
1	Mitsubishi	Green	$5000	18
2	Mitsubishi	Green	$10000	14
3	Mitsubishi	Green	$15000	4
4	Mitsubishi	Black	$5000	15
5	Mitsubishi	Black	$10000	13
6	Mitsubishi	Black	$15000	10
7	Ford	Green	$5000	16
8	Ford	Green	$10000	8
9	Ford	Green	$15000	3
10	Ford	Black	$5000	17
11	Ford	Black	$10000	9
12	Ford	Black	$15000	7
13	Toyota	Green	$5000	11
14	Toyota	Green	$10000	5
15	Toyota	Green	$15000	1
16	Toyota	Black	$5000	12
17	Toyota	Black	$10000	6
18	Toyota	Black	$15000	2

With the respondent's preferences given, coding of the combinations and rating must be performed. In dummy-variable coding a binary representation is used to form the regression problem. For the presence of an attribute level in a combination, a '1' is used and a '0' symbolizes the absence of an attribute level. The ending result is a n x m table, where n is the total number of attribute levels and m is the number of combinations, containing only ones and zeros in the left section and the far right column depicting the rating of the respondent as shown in Table 4.3.

Table 4.3: Dummy-Variable Binary Representation

Number	***Make***			***Color***		***Price***			***Ranking***
	M	F	T	G	B	$5000	$10000	$15000	
1	1	0	0	1	0	1	0	0	18
2	1	0	0	1	0	0	1	0	14
3	1	0	0	1	0	0	0	1	4
4	1	0	0	0	1	1	0	0	15
5	1	0	0	0	1	0	1	0	13
6	1	0	0	0	1	0	0	1	10
7	0	1	0	1	0	1	0	0	16
8	0	1	0	1	0	0	1	0	8
9	0	1	0	1	0	0	0	1	3
10	0	1	0	0	1	1	0	0	17
11	0	1	0	0	1	0	1	0	9
12	0	1	0	0	1	0	0	1	7
13	0	0	1	1	0	1	0	0	11
14	0	0	1	1	0	0	1	0	5
15	0	0	1	1	0	0	0	1	1
16	0	0	1	0	1	1	0	0	12
17	0	0	1	0	1	0	1	0	6
18	0	0	1	0	1	0	0	1	2

The above data has a linear dependency which represents a complication in the analysis. Multiple regression analysis is used to determine the part-worth values from the above data. In this analysis no independent variable can be perfectly predictable from the value of any other independent variable or combination of variables [49]. The linear dependency is resolved by omitting one column of data from each attribute. The omission of one of the levels implicitly denotes an attribute level as a reference (i.e. part-worth of zero) for the other levels. The specific level does is not important and does not affect the outcome of the regression.

The rating/ranking data is fit to a regression model of the form,

$$y = b_0 + b_1 x_1 + b_2 x_2 \ldots\ldots b_n x_{n+e} \tag{4.3}$$

where y is the rating/ranking value, b_0 is an intercept term, $b_1, b_2, \ldots, b_n$ are the part worth utilities of the $x_1, x_2, \ldots x_n$ attribute levels, and e is an error term. There are different criteria for the use of a rating scheme for preference elicitation or ranking. In the case of rating, Equation (4.2) may be used directly where y is the given rating from the DM. In the case where ranking is used to measure the customer's preference a logit recode of the given ranking value is required. The reason is because Ordinary Least Squares regression methods are not appropriate for conjoint data consisting of rank orders [50-51]. This is due to the different between the representation of a rating and a ranking. In a rating the data is scaled so that real differences in combinations are communicated by the arithmetic differences in their value. In other words, the difference between a rating of a 1 and 2 is the same as the different between a rating of 9 and 10. In rankings, the same assumption cannot be true. For instance, a combination with a ranking of 4 is necessarily twice as preferred as the combination ranked 2.

To perform the logit coding[46], a probability value, p, of each ranking is,

$$p = \frac{\bar{y} - min + 1}{max - min + 2} \quad (4.4)$$

where $\bar{y}$ is the ranking value given by the respondent and *min* and *max* are the minimum and maximum ranking value used. The *p* value is then used to calculate the logit coded ranking value, y_L,

$$y_L = \ln\left(\frac{p}{1-p}\right) \quad (4.5)$$

This recode is performed for each ranking value and used to evaluate Equation (4.1) for the regression problem. The logit coding is a transformation of the ranking values into a scaled value in which it is appropriate to use an Ordinary Least Squares regression method such as multiple regressions.

When the Dummy-Variable regression is conducted, there is a possibility to get very different part-worth utilities depending on the value of the intercept term (i.e. zero or non-zero). This can be a critical issue since the intercept term may represent a reference point for the each attribute level. This is the main reason of considering Effects Coding[47] as an alternative to Dummy Variable Regression for determining the part-worth utilities due to the possibility of the statistically significant intercept term b_0 as shown in Equation (4.1).

For Effects Coding, the reference level of each attribute is assigned a value of '-1' for all combinations as opposed to removing the level completely as in Dummy-Variable Regression. The binary matrix is formed in the same manner by representing the presence of an attribute level in a combination with a '1' and the absence of a level with a '0'. The ranking/rating data from the DM is represented in the far right column of the binary matrix. The presence of the ‘-1’ in Effects Coding helps to define the reference level as the negative sum of the estimated coefficients (i.e. the part-worth values of the other levels). In other words, the reference point is internalized in the *b* variables in Equation (4.3) as opposed to being carried over on the intercept term.

The solution to the multiple regression analysis minimizes the sum of squares of the errors over all observations. A regression equation is typically solved for each respondent. Thus it is required to evaluate a minimum of one combination per parameter for an accurate estimate of the part-worth utilities [52]. However, if only the minimum is done then there is no room to account for respondent error so traditionally more combinations are assessed to provide a better approximation.

Once the part-worth utilities are obtained for each level of each attribute, the importance of each attribute is calculated. This process is performed to calculate the relative importance of each attribute. The importance is the difference an attribute contributes to the total utility of a product. As shown in Table 4.4, the difference is the range in the utilities for each attribute. Percentage values, that add up to 100%, are calculated from the ranges. For the given example, Price has an importance of 58.044%, Make has an importance of 36.06% and Color has 5.898%.

Table 4.4: Calculation of Attribute Importance

Attribute	*Level*	*Part-Worth Utility*	*Attribute Utility Range*	*Attribute Importance*
Make	Mitsubishi Ford Toyota	1.7957 1.1922 0	1.7957-0=1.7957	(1.7957/4.98)*100=**36.06%**
Color	Green Black	0 0.2937	0.2937-0=0.2937	(0.2937/4.98)*100=**5.898%**
Price	$5,000 $10,000 $15,000	2.8906 1.3393 0	2.8906-0=2.8906	(2.8906/4.98)*100=**58.044%**
			Total Utility=4.98	

CHAPTER 5. IMPROVING QUALITY DURING PRODUCT DESIGN

5.1 Importance of quality

Every manufactured product has characteristics that determine its performance. For example, a car's performance is measured by mileage and acceleration. A clutch's performance is measured by torque capacity. These are of concern to customers at time of purchase [53-54].

Quality control, quality assurance and total quality management all deal with reducing the variation in the product's main characteristic. The quality of a system is inversely proportional to the variation in its main characteristic [55]. The loss a customer sustains when a product deviates from its normal functioning is known as the quality loss.

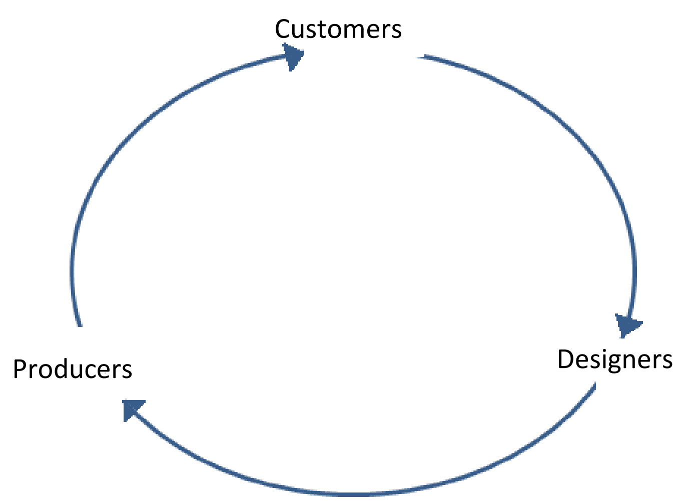


Figure 5.1: Quality Circle [54]

Quality is determined by the customer. This is indicated in the quality circle in Figure 5.1. Customers indicate their preferences through past purchases of similar products. Designers obtain the needs, wants and expectations from a particular product through information from the customer. These are translated into product specifications, which include drawings, dimensions, materials, tolerances, processes, tooling and gaging. Using this information, the product is fabricated and delivered to the customer via marketing channels. The product needs to arrive in the right quantities, in the right manner, at the right place and provide correct functioning for the correct period of time. Customer feedback to the designers is revealed through surveys, number of products sold and complaint rate [54].

5.2 Taguchi Loss Function

Traditionally, industries measure quality by the defect rate. Defects are identified through quality inspection of products, where the characteristic output is examined to ensure that it falls within a certain range of output value. For example, when a door is opened, one can keep it open by placing a stopper in front of it. The force required to close the door is an important design requirement for the customer. If the force is too high or the door is too heavy, a weaker individual may not be able to do it. If the force is too small, a gust of wind could cause it to close, and the customer will want to replace the door. There needs to be a range in the engineering specifications for the force values required to close the door assembly. A range is required, since doors are used for different purposes and are made in different dimensions. As long as the force is within the range, the customer is satisfied. A view of this philosophy is shown in Figure 5.2.

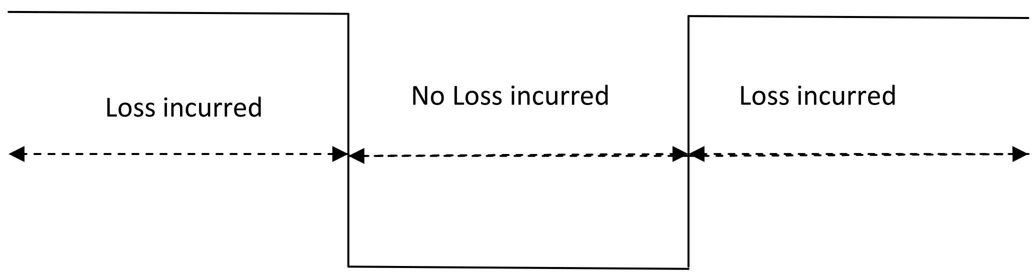


Figure 5.2: Goalpost View of Losses [56]

This philosophy is known as the goalpost philosophy since it is similar to the utilization of goalposts in football. If the ball passes between the posts, it is considered a successful shot. The actual point at which the ball crosses the post does not matter. Similarly, if the ball misses the target, it is unsuccessful [56]. Most of American industry has been managed by the goalpost philosophy since the Industrial Revolution. This emphasizes the importance of meeting specifications. Quality-control inspectors measure product characteristics to determine if they meet requirements, or lie within the 'goalpost' [54].

The Quality loss function is based on the work of an electrical engineer, Genichi Taguchi. He rejects the traditional goalpost philosophy. He asks the fundamental question: Is there a vital difference in quality when a characteristic lies just inside the allowed range versus one that lies just outside the range? He asserts that the difference is insignificant and there needs to be better methods to improve product quality.

He emphasizes that efforts are better spent trying to minimize variability of the characteristic around a single value, instead of trying to keep it between a range of values. Departures from an optimal value represent a loss to society, and minimizing the value of loss reduces the loss to society. This approach offers a method of testing designs and process parameters with a minimum number of test specimens [54].

The Taguchi loss function quantifies the variation present in the performance characteristic. If the door functions perfectly with a closing force of 50 N, the Taguchi Loss function calculates the variation of the force from 50 N, due to tolerances in the inputs. This is indicated in Equation 5.1 and shown in Figure 5.3.

$$L = k(y - m)^2 \tag{5.1}$$

where k is the Taguchi Loss constant, m is the target value(in this case 50 N) and y is the output.

The Taguchi constant value, k, is determined using cost of repairs for previously manufactured doors. If the cost of fixing a door that deviates from the target by 10 N is \$10, the value of k is calculated as:

$$\$10 = k\,(10)^2$$

$$k = 0.1$$

Equation (5.1) now changes to:

$$L = 0.1(y - m)^2 \tag{5.2}$$

The loss L is generally optimized to a minimum under the design parameters for the problem (in this case, the dimensions of the door). The above case is an example of a 'nominal is best' loss function. A nominal value is declared the best possible output, and any deviation from the nominal is considered a loss.

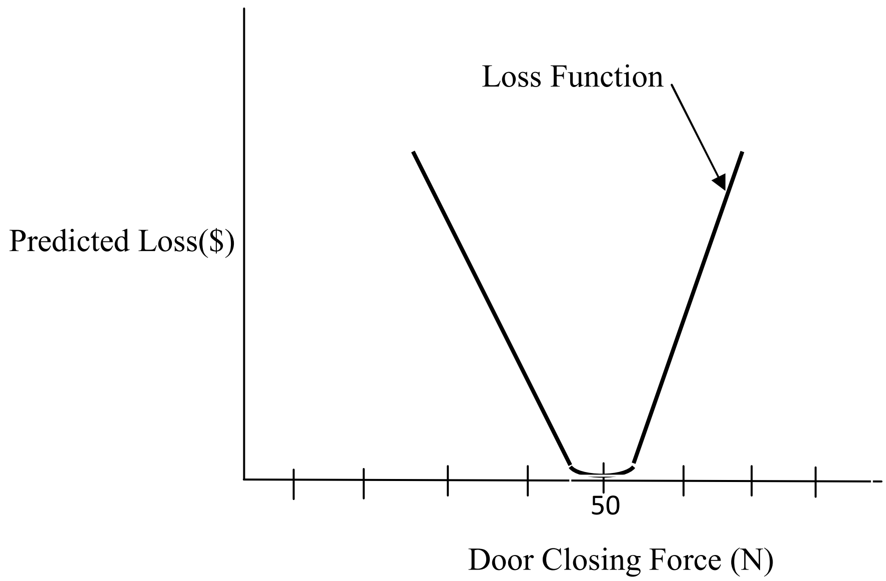


Figure 5.3: Taguchi's Quality Loss Function [56]

The use of Taguchi's quality loss function has proved to be an effective method of improving quality in many US and European firms over the last fifteen years [57-60].

5.3 Other forms of Taguchi loss function

In certain cases, there is no optimal value (target value) for the product characteristic. Certain characteristics have their best value as the highest possible value and certain ones have their best value as the lowest possible value. A good example for a lower-is-better characteristic is the waiting time for the delivery of a product to a customer[54]. If the company reveals that it will take a few days for the product to arrive, there is a feeling of loss. The longer the wait is, the larger the loss. Another example of lower is better includes friction loss. The loss function for a lower is better characteristic is shown in Figure 5.4.

Mileage, energy output, efficiency are examples of higher is better characteristics. The loss function for such a characteristic is shown in Figure 5.5.

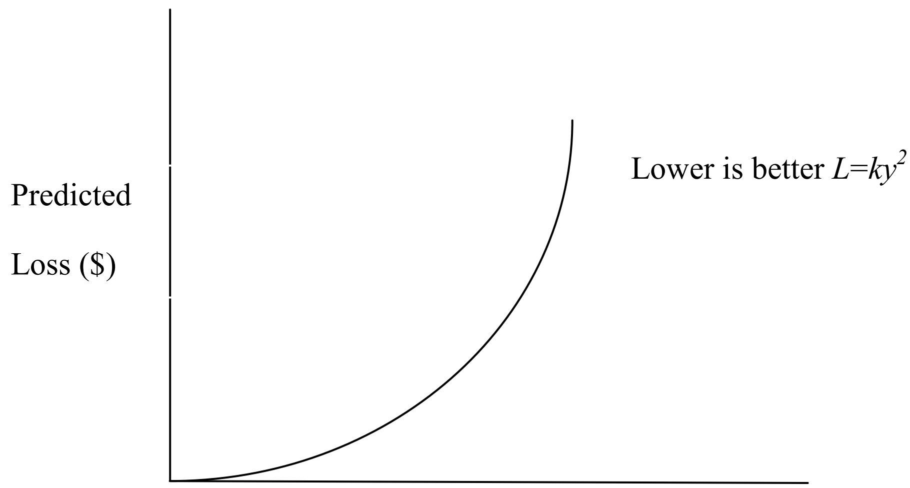


Figure 5.4: Lower is better, $L=ky^2$

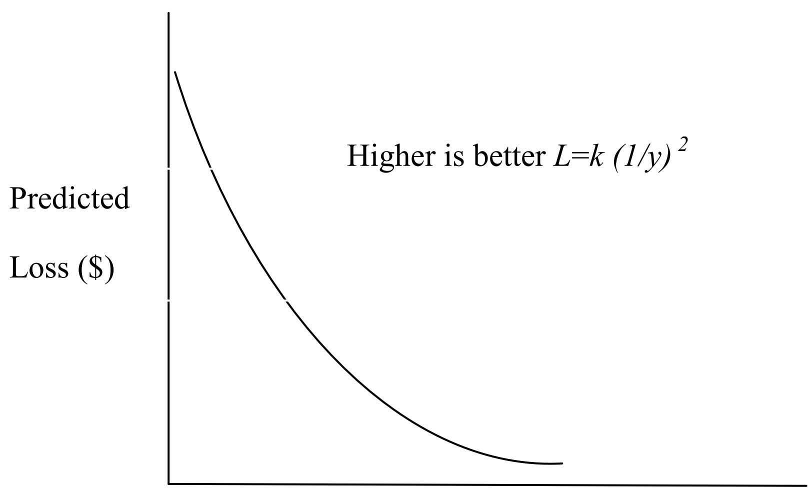


Figure 5.5: Higher is better, $L=k\ (1/y)^2$

CHAPTER 6. PROPOSED METHOD

To improve the Fuzzy Comprehensive Evaluation (FCE) procedure, there needs to be answers to the research questions at the end of Chapter 2. An integrated method is proposed that utilizes Conjoint Analysis to calculate the weighted importance values for the fuzzy factors. The Taguchi Quality Loss method is incorporated into the cost equation to obtain robust output for the assembly.

In the existing method (FCE), it is difficult for the expert to accurately estimate the importance of each factor. If a survey is conducted asking the experts to rate the importance of each factor, the experts may respond only with higher ratings. This is because all factors are considered important in determining machinability. This results in insufficient data for determining importance factors: skewed data that has little differentiation between factors. Even though it is easy for respondents to complete this type of survey, these ratings are not very meaningful. In real life it is not possible to get the best of each attribute. One has to make the trade-offs and concessions [9]. Also, individual attributes in isolation are perceived differently than the combination of levels in those attributes. It is better if a respondent is provided with a list of combinations of each attributes and asked to rank them. This kind of survey, becomes impractical, however, when there is a substantially large number of combinations.

6.1 Introduction of Conjoint Analysis (CA) procedure

To improve the accuracy of the evaluation in the FCE process, the proposed framework introduces CA to determine the weighted importance vector *I*. The four fuzzy factors are considered to be the attributes and the grades of each factor are the levels of each attribute. The CA allows ranking of a subset of the possible combinations of levels of each attribute to determine the relative importance of each attribute. A regression analysis of the ranking is done to determine the part-worth utility of each level. These values are used to determine the overall utility of each attribute. This method is efficient since all the combinations need not be considered for the ranking. The modified framework, that incorporates conjoint analysis, is shown in Figure 6.1

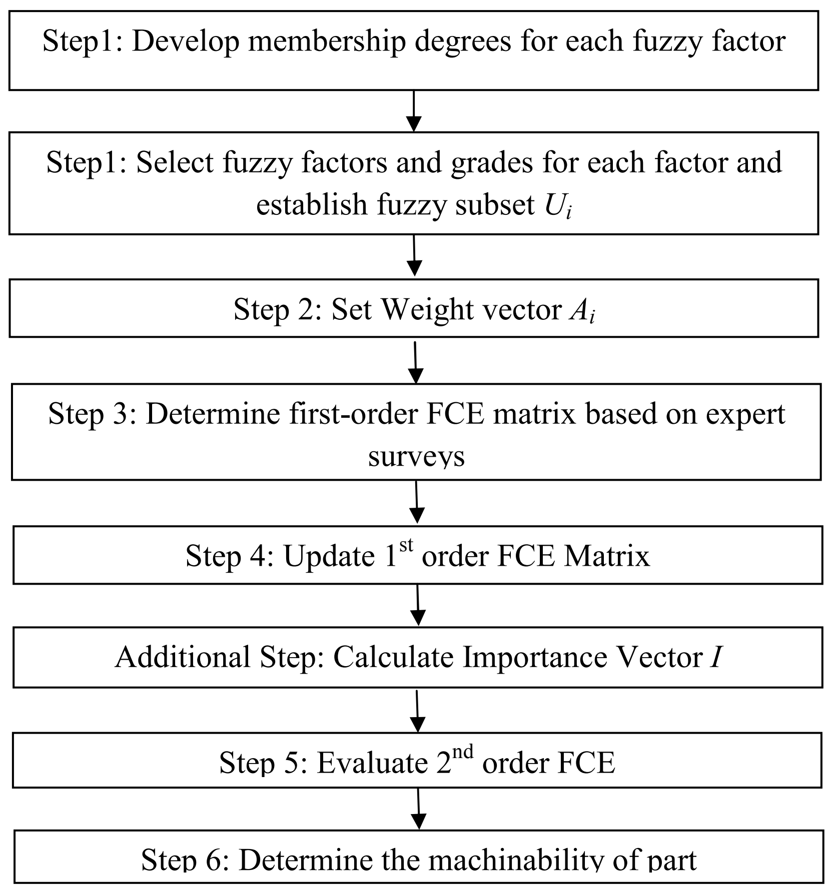


Figure 6.1: Modified Fuzzy Comprehensive Evaluation Framework

As shown in Figure 6.1, the additional CA step has been introduced before calculating the second order FCE. The CA procedure eliminates the arbitrary assignment of the importance vector of factors in the FCE. Once the machinability values are calculated for each component using the FCE and CA, the tolerance allocation is conducted.

6.2. Reducing variability in output through Taguchi function

The cost function, used to allocate tolerances, takes into account the machinability of each assembly part. Cost is minimized while taking into account the four fuzzy factors that influence machinability, Dimension Size (DS), Geometric Structure (GS), Material Machinability (MM) and process Accuracy (PA). The Fuzzy comprehensive Evaluation (FCE) procedure, described in Chapter 3, is used to calculate the cost function for cost due to machinability, C_M.

$$C_M = C_0 + \sum_{i=1}^{n} \frac{\psi_i}{tol_i} \tag{6.1}$$

where C_0 is the setup cost, Ψ_i is the comprehensive factor of part i in the assembly and tol_i is the tolerance of part i in the assembly.

In a lot of assemblies, the tolerance plays a critical role in the final performance. A large variation in performance values caused by a certain tolerances allocated could damage the machine. To decrease variation in the output performance, the Taguchi Quality loss function is introduced to the framework. It calculates the costs, C_L, associated with variation of the performance from its expected measure.

$$C_L = k(y - m)^2 \tag{6.2}$$

where y is the value of the performance measure at a certain assembly tolerance level and m is the target performance measure. k is the Taguchi loss constant. The cost equation in the proposed framework incorporates both costs due to machinability, C_M, and costs due to quality loss, C_L. This is described in Equation (6.3) and (6.4).

$$C = C_M + C_L \tag{6.3}$$

$$C = C_0 + \sum_{i=1}^{n} \frac{\psi_i}{tol_i} + k(y-m)^2 \tag{6.4}$$

The proposed framework for tolerance allocation in this thesis is summarized in Figure 6.2. In certain assemblies, the allocated tolerances do not affect the performance of the assembly. Then, C_L, in Equation 6.3 is set to zero before optimizing the total cost C.

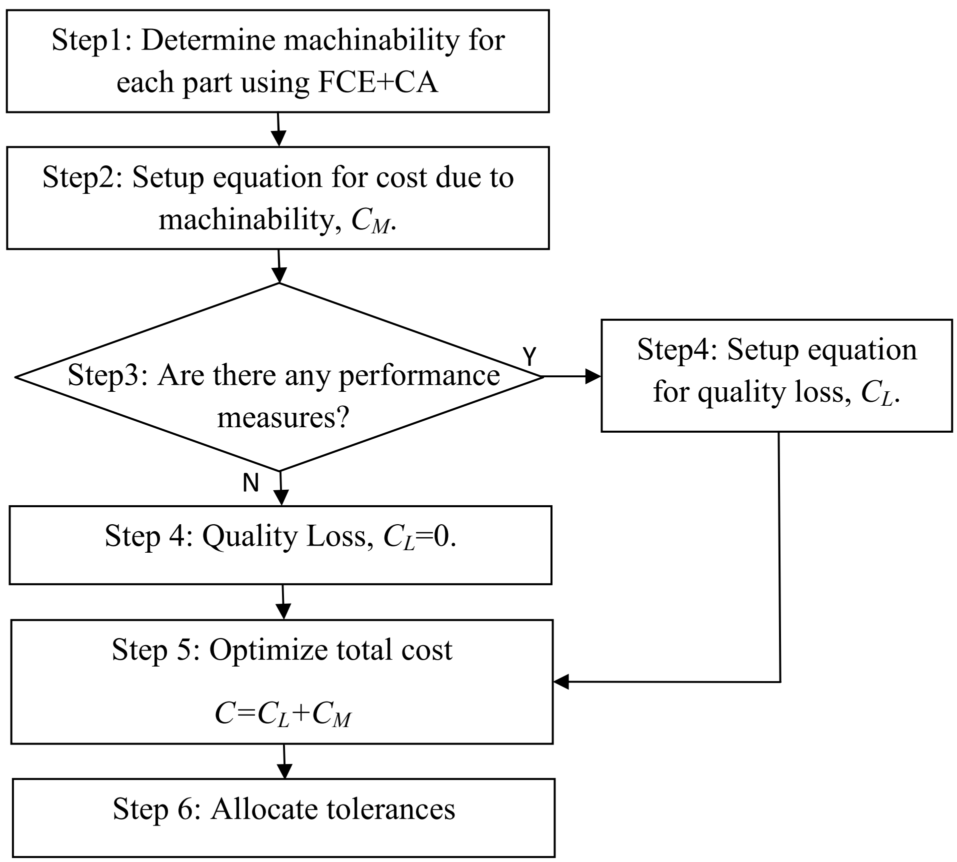


Figure 6.2: Proposed Framework for Tolerance Allocation

6.3 Validation of proposed method

The proposed method is applied to three different engineering applications in Chapter 7. Firstly, it is used to allocate tolerances for parts of a simplified clutch assembly. In the second application, the method is applied for tolerance allocation of an O-ring seal assembly in an accumulator. Finally, the tolerances in a Power Generating Shock Absorber (PGSA) are allocated to minimize cost and increase robustness in output performance.

It is important that the utility of Conjoint Analysis (CA) and Taguchi's quality Loss function is validated. For each of the three engineering applications, two validation procedures are performed:

1. Validating Conjoint Analysis through Cross Validation

Cross Validation is a technique for determining the effectiveness of a predictive model. Several methods of cross validation that have been used in the past [61].

k-fold cross validation is an accurate, computationally inexpensive method of Cross Validation. The original data is randomly broken into k subsections. A model is then built using (k-1) subsections as training data, and the remaining set as validation data. This process is repeated k times, called folds, until every subsection has been used as validation data. This method is more accurate than test set cross validation, for every data point is used as both a training set and a validation set. To provide validation, two cases are examined where $k = 3$.

To provide sufficient validation, two cases are examined:

Case A: Tolerance Allocation with CA procedure

The data is randomly broken into three subsections: *a, b,* and *c*. For the first fold, subsections *a* and *b* will be the training set, and section *c* will be the test set. The proposed framework in Chapter 6 is applied to the training set, and the weighted importance vector is calculated. The procedure is then repeated twice more, using subsections *a* and *c* as the training set and using subsections *b* and *c* as the training set, calculating the weighted importance vector for both cases.

When all three importance vectors have been calculated, they are individually used to calculate three sets of tolerances, using the proposed framework. RMS_e is then calculated for each case using equation 6.5.

$$RMS_e = \sqrt{\frac{\sum_{i=1}^{k}(tol_{i1} - tol_{i2})^2}{k}} \tag{6.5}$$

where tol_{i1} is the tolerance of part *i* calculated using the complete data set, tol_{i2} is the tolerance of part *i* calculated using the training data set, *k* is the number of parts in the assembly. To derive total error, the mean of the three RMS errors is taken.

$$RMS_{e,total} = RMS_{e,ab} + RMS_{e,bc} + RMS_{e,ca} \tag{6.6}$$

Where $RMS_{e,i}$ is the RMS error of training set *i*.

Case B: Tolerance Allocation without CA procedure

100 experts vote on which factor they deem the most important. The data is randomly divided into three subsections, a, b and c. For the first fold, subsections *a* and *b* will be the training set, and section *c* will be the test set. A weighted importance vector is derived based upon the percentage of experts preferring each factor. The procedure is repeated twice, once using training set a and another time using training set b. When the

three weighted importance vectors have been calculated, the tolerances are allocated and RMS error is derived, using Equation 6.5. The total RMS error is calculated using Equation 6.6.

For example, if 30% vote for factor DS, 30% for factor GS, 30% for factor MM and 10% for factor PA, the vector *I* is [0.3 0.3 0.3 0.1]. In the cross-validation process, 33% of the experts become the test data set. The rest of the experts become the training data set. The training data set is used to calculate a new importance vector. If 10 out of the 30 experts that were on the test set voted for factor DS, 10 for factor GS, 10 for factor MM and 3 for factor PA, then the new importance *I* is [20/67 20/67 20/67 7/67]. The tolerances are allocated using both importance factors, and the RMS error is calculated using Equation 6.5. The total RMS error is calculated using Equation 6.6.

If the RMS error value obtained using Case A is smaller, the use of Conjoint Analysis has been validated.

2. Validation of Taguchi's Quality Loss Function

The Taguchi method is introduced to reduce variation in the output performance in engineering applications where the tolerance affects output performance. To provide sufficient validation of the method, two cases are examined.

Case A: Tolerance Allocation without Taguchi method

The quality loss function is set to zero (C_L=0) before performing the final optimization. The output measure is calculated using the allocated tolerances, and the deviation from the expected performance measure is obtained using:

$$Err = abs(k - m) \tag{6.7}$$

where *m* is the expected performance value and *k* is the obtained performance value.

Case B: Tolerance Allocation using Taguchi method

The quality loss function is included in the cost equation, as in Equation 6.4. The final tolerances are allocated and the output performance is calculated using the allocated tolerance. The deviation from the expected performance measure is obtained using Equation (6.7).

If the deviation in Case B is smaller, the use of Taguchi quality loss function has been validated.

CHAPTER 7. APPLICATION OF PROPOSED FRAMEWORK

7.1 Friction Clutch Assembly

The design and manufacturing processes of heavy machinery which is manually operated requires the highest standards of quality and reliability. Machines that are large, mass produced, and common all over the world must be designed with an intrinsic safety value. Failure of components, either from poor design or discounted manufacturing practices could lead to loss of life, and tarnishing of a brand name. However, over-engineering the safety aspect of a component can be an obstacle during its journey from a blueprint to the production line. Successful companies strike a balance between performance, safety and costs.

One example that illustrates this balance is friction clutch design in automotives. The design of the clutch has survived the test of time due to a combination of simplicity of design but broad range of applications. It is a mechanism for transmitting rotational motion from one rotational shaft to another. Normally, one rotating shaft is attached to an engine and the other is attached to the actuating device. Figure 7.1 shows a simplified model of the assembly cross section. The pressure plate is used to engage/disengage the clutch from the flywheel when the vehicle is in gear/out of gear, respectively. When the clutch pedal is depressed by the driver, the pressure plate is released from the clutch and the flywheel is disengaged. Once the driver chooses a gear, the clutch is released. The pressure plate is pressed against the clutch again, engaging it with the flywheel. This returns the transmission unit to normal functioning.

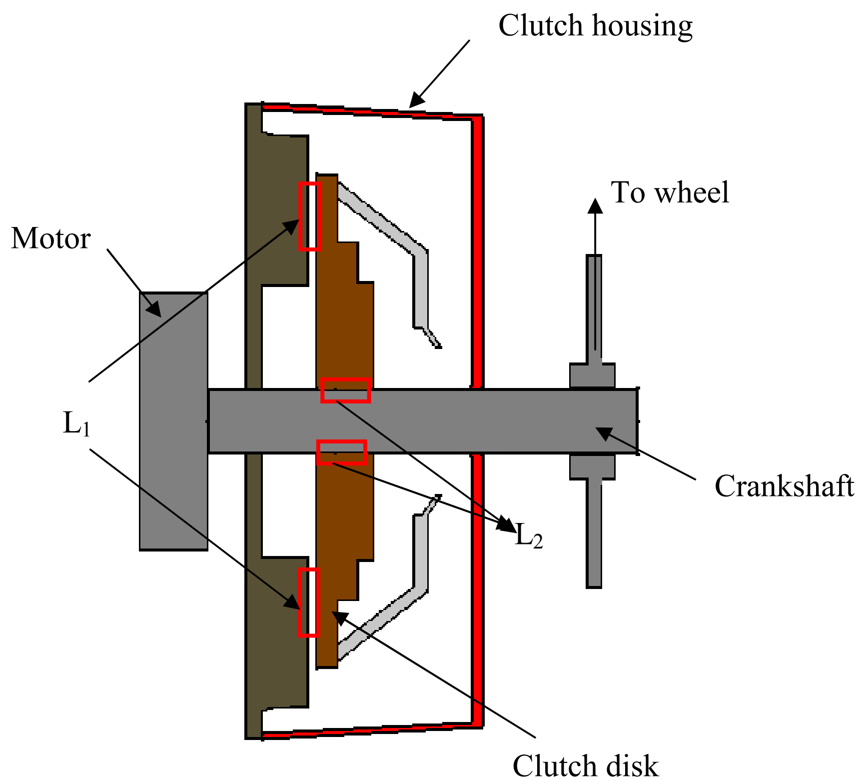


Figure 7.1: Cross-section of Clutch Assembly

Proper functioning of the clutch is vital for performance of the vehicle. Improper tolerance allocation can result in malfunctioning of the clutch which is a detriment to the safety of the passenger. Tolerance allocation determines the tolerance of the parts ensuring that the tolerances of the clearance locations in the assembly are within the functioning limits. When engaged, improper tolerance allocation causing interference in L_1 (location shown in Figure 7.1) can cause the clutch to depress into the flywheel, increasing wear and reducing life of the clutch assembly. It is also possible for the clutch to not be completely engaged with the flywheel (due to a gap in L_1), causing weak power transmission and poor driving performance. Thus, it is important that the clutch is toleranced tightly. However, if the tolerance is too tight, the cost of the component makes the product uncompetitive in the market. An optimal tolerance value is required to allow for proper functioning of the clutch while maintaining a competitive manufacturing cost.

The dimensioning used for tolerance allocation is shown in Figure 7.2 and the corresponding variables are listed in Table 7.1. The two air gaps are assumed to be constant (zero tolerance) and subtracted from the total thickness and housing length. The proposed method, described in Chapter 6, is applied to the tolerance allocation problem of clutch assembly. The grades for each of the four factors to be used in this problem are summarized in Table 7.2.

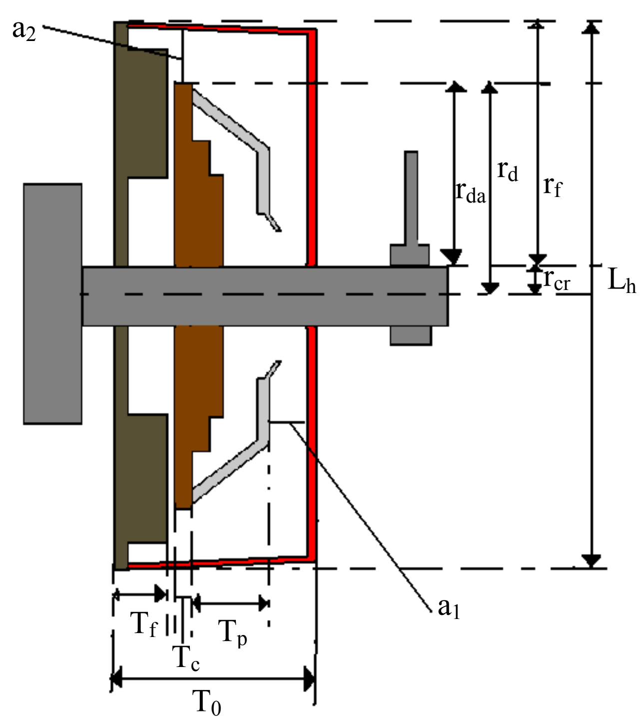


Figure 7.2: Clutch Assembly Dimension

Table 7.1: Variable Descriptions and Dimensions

Variable	Description	Value(cm)
r_f	Flywheel radius	13.2
r_{cr}	Crankshaft radius	2.1
r_d	Clutch disk radius	11.3
r_{da}	Clutch disk annular radius	9.2
T_f	Flywheel thickness	3.9
T_p	Pressure plate thickness	5.5
T_c	Clutch disk thickness	0.85
T_0	Total thickness	11.25
L_h	Housing length	26.0
a_1	Airgap 1	1.0
a_2	Airgap 2	0.5
$T_{0a}=T_0-a_1$	Total Thickness minus airgap	10.25
$L_{ha}=L_h-2a_2$	Housing length minus airgap	25.0

Table 7.2: Grade Divisions of Fuzzy Factors for Clutch

	Grade 1	Grade 2	Grade 3	Grade 4
U_1 (DS)	~0 cm	~10 cm	~20 cm	~30 cm
U_2 (GS)	Easy to manufacture	Hard to manufacture	-	-
U_3 (MM)	Poor	Medium	Good	-
U_4 (PA)	Poor	Medium	Good	-

7.1.1 Membership degrees elicitation

To conduct the FCE process, the fuzzy subsets of the four factors, DS, GS, MM and PA (with fuzzy subsets U_1 to U_4) are determined first for each assembly part. The first factor, Dimension Size (DS) are determined based on the dimension values of each part. There is no need to elicit membership degrees for the values. The fuzzy subset for DS (U_1) is determined based on the value in Table 7.1 and listed in Table A.1 in the Appendix.

Membership degrees for GS, MM and PA are determined using the rank ordering method described in Chapter 3.2 Step 1. 100 experts are asked to compare two grades of each factor as to which one of them is better suited to describe the fuzzy factor for the part. This comparison is done for all combinations of grade pairs. A percentage preference is established for each grade which is the membership degree value of the grade for the assembly part.

For the fuzzy factor GS, the fuzzy subset (U_2) is listed in Table A.2 in the Appendix. The pressure plate is hardest to machine due to its irregular shape. The crankshaft is easiest to machine since it has the simplest geometry. The overall thickness has a GS value that is the same as the part with the highest GS value.

For the fuzzy factor MM, the fuzzy subset (U_3) is listed in Table A.3 as the Appendix. The experts determine the MM value based on the material malleability. A component, whose material has higher malleability, has higher MM. The typical clutch disk is made of non-asbestos based friction material with high copper content. It is assumed to have the malleability properties of copper to simplify creation of the membership degrees. The pressure plate and crankshaft are made of steel; housing and flywheel are made of cast iron. The copper based material is the most malleable metal while cast iron is the least malleable[62]. The list of materials is shown in Table 7.3.

Table 7.3: Clutch Assembly Materials

Assembly part	**Material**
Flywheel	Cast iron
Housing	Cast iron
Pressure plate	Steel
Crankshaft	Steel
Clutch Disk	Copper based friction material

For the fuzzy factor PA, the fuzzy subset (U_4) is listed as Table A.4 in the Appendix. The experts rank order the subsets for each part based on how often the part is in contact with other parts of the system. The more contact that occurs for a part while engaging and disengaging the clutch, the higher the required PA is for proper functioning of the system. For this reason, the clutch disk requires higher PA than the housing. The PA membership value for the overall thickness is the same as that for the part with highest accuracy.

Once the fuzzy subsets are determined, the next step is to obtain the first order FCE matrix, as in Equation 3.4. The matrices for each of the factors are evaluated by experts as described in Section 3.2 Step 4. It is determined based on the experience of experts. In the matrix, each fuzzy factor is classified into ten machinability levels, ranging from *A-J*. Machinability level *A* is the level in which the part is easiest to machine. Machinability level *J* is the level in which the part is hardest to machine. A panel set of industry experts are asked to vote for the most appropriate machinability level for each grade.

The percentage values are scaled between 0-1, and included in the columns of the 1st order FCE matrix. The number of rows in the matrix is the same as the number of grades of each factor. The number of columns is the number of machinability levels. Consider the factor DS. The lowest grade of DS for the component is approximately 0 cm. It is considered easiest to machine due to lower material cost. 90% of the respondents vote for Level A, 5% level B and 5% level C. Before scaling, the first row of the matrix for dimension size is:

$$R_{DS,1} = [0.9\ 0.05\ 0.05\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$$

The first row is obtained, after scaling to value between 0 to 1 (dividing by maximum value 0.9);

$$R_{DS,1} = [1\ 0.06\ 0.06\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$$

After completing a similar process for each of the other grades, the first order FCE matrix for the DS, R_{DS}, is obtained as:

$$R_{DS}=\begin{bmatrix} 1 & 0.06 & 0.06 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 1 & 0.1 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 1 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 1 & 0.8 \end{bmatrix}$$

A similar procedure is applied to obtain the 1st order FCE matrices for the other factors,

$$R_{GS}=\begin{bmatrix} 0.2 & 1 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0.3 & 1 \end{bmatrix}$$

$$R_{MM}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.11 & 0.11 & 1 \\ 0 & 0 & 0 & 0 & 0.1 & 0.2 & 1 & 0.05 & 0.05 & 0 \\ 1 & 0.3 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$R_{PA}=\begin{bmatrix} 1 & 0.2 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0.2 & 1 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0.3 & 1 \end{bmatrix}$$

where R_{GS}, R_{MM} and R_{PA} are the 1st order FCE matrices for the factors GS, MM and PA respectively.

7.1.2 Determining Importance Factors Using Conjoint Analysis

The CA method, described in Chapter 4, is applied to evaluate the importance factors, *I*. The four grades, DS, GS, MM, and PA are the four attributes and they have four, two, three and three levels respectively. Thus, there are 72 combinations. To make ranking easier for the experts, a 1/3 fractional factorial is taken, resulting in a total of 24 combinations[45]. A fractional factorial design will take an adequate fraction of combinations, with as little an effect on the final combination as possible. The set of 24 combinations that are used for the Conjoint Analysis are shown in Table 7.4.

Table 7.4 Fractional Factorial Design for Friction Clutch Application

Combination	***DS***	***GS***	***MM***	***PA***
1	~0 cm	Easy To Manufacture	Medium	Poor
2	~0 cm	Easy To Manufacture	Poor	Medium
3	~0 cm	Easy To Manufacture	Good	Good
4	~0 cm	Hard to manufacture	Good	Medium
5	~0 cm	Hard to manufacture	Medium	Good
6	~10 cm	Easy To Manufacture	Poor	Poor
7	~10 cm	Easy To Manufacture	Medium	Medium
8	~10 cm	Hard to manufacture	Good	Poor
9	~10 cm	Hard to manufacture	Good	Medium
10	~20 cm	Easy To Manufacture	Poor	Poor
11	~20 cm	Easy To Manufacture	Medium	Medium
12	~20 cm	Easy To Manufacture	Poor	Good
13	~20 cm	Hard to manufacture	Medium	Poor
14	~20 cm	Hard to manufacture	Poor	Medium
15	~30 cm	Easy To Manufacture	Good	Good
16	~30 cm	Easy To Manufacture	Poor	Medium
17	~30 cm	Hard to manufacture	Good	Medium
18	~30 cm	Hard to manufacture	Good	Good
19	~0 cm	Hard to manufacture	Medium	Poor
20	~10 cm	Easy To Manufacture	Good	Poor
21	~10 cm	Hard to manufacture	Medium	Good
22	~20 cm	Hard to manufacture	Medium	Medium
23	~30 cm	Easy To Manufacture	Medium	Medium
24	~30 cm	Hard to manufacture	Medium	Poor

The next step is to gain the respondent's preferences for each of the above combinations through customer surveys, computer programs, or elicitations from designers. The respondent for this application is an expert in the field of manufacturing the friction clutch. The subjective data elicited are based on the attributes and preferences on the respondent. Therefore the suggested final design will represent the preferred design as pertains to the expert(s) giving the rating/ranking data. This is shown in Table 7.5.

Table 7.5: Expert Rating of Attribute Combinations

Comb.	***DS***	***GS***	***MM***	***PA***	***Rank***
1	~0 cm	Easy To Manufacture(E)	Medium(M)	Poor(P)	21
2	~0 cm	Easy To Manufacture(E)	Poor(P)	Medium(M)	22
3	~0 cm	Easy To Manufacture(E)	Good(G)	Good(G)	24
4	~0 cm	Hard to manufacture(H)	Good(G)	Medium(M)	16
5	~0 cm	Hard to manufacture(H)	Medium(M)	Good(G)	12
6	~10 cm	Easy To Manufacture(E)	Poor(P)	Poor(P)	11
7	~10 cm	Easy To Manufacture(E)	Medium(M)	Medium(M)	20
8	~10 cm	Hard to manufacture(H)	Good(G)	Poor(P)	6
9	~10 cm	Hard to manufacture(H)	Good(G)	Medium(M)	8
10	~20 cm	Easy To Manufacture(E)	Poor(P)	Poor(P)	10
11	~20 cm	Easy To Manufacture(E)	Medium(M)	Medium(M)	18
12	~20 cm	Easy To Manufacture(E)	Poor(P)	Good(G)	17
13	~20 cm	Hard to manufacture(H)	Medium(M)	Poor(P)	4
14	~20 cm	Hard to manufacture(H)	Poor(P)	Medium(M)	2
15	~30 cm	Easy To Manufacture(E)	Good	Good(G)	23
16	~30 cm	Easy To Manufacture(E)	Poor(P)	Medium(M)	3
17	~30 cm	Hard to manufacture(H)	Good(G)	Medium(M)	5
18	~30 cm	Hard to manufacture(H)	Good(G)	Good(G)	19
19	~0 cm	Hard to manufacture(H)	Medium(M)	Poor(P)	13
20	~10 cm	Easy To Manufacture(E)	Good(G)	Poor(P)	15
21	~10 cm	Hard to manufacture(H)	Medium(M)	Good(G)	9
22	~20 cm	Hard to manufacture(H)	Medium(M)	Medium	7
23	~30 cm	Easy To Manufacture(E)	Medium(M)	Medium(M)	14
24	~30 cm	Hard to manufacture(H)	Medium(M)	Poor(P)	1

The Dummy Variable Regression table is shown in Table 7.6. The y-values are calculated using Logit coding in Equation (4.4-4.5).

Table 7.6: Dummy-Variable Binary Representation

Comb.	***DS(~cm)***				***GS***		***MM***			***PA***			***Rank***	***Y***
	0	10	20	30	E	H	P	M	G	P	M	G		
1	1	0	0	0	1	0	0	1	0	1	0	0	21	1.658228
2	1	0	0	0	1	0	1	0	0	0	1	0	22	1.992430
3	1	0	0	0	1	0	0	0	1	0	0	1	24	3.178053
4	1	0	0	0	0	1	0	0	1	0	1	0	16	0.575364
5	1	0	0	0	0	1	0	1	0	0	0	1	12	-0.08004
6	0	1	0	0	1	0	1	0	0	1	0	0	11	-0.24116
7	0	1	0	0	1	0	0	1	0	0	1	0	20	1.386294
8	0	1	0	0	0	1	0	0	1	1	0	0	6	-1.15268
9	0	1	0	0	0	1	0	0	1	0	1	0	8	-0.75377
10	0	0	1	0	1	0	1	0	0	1	0	0	10	-0.40546
11	0	0	1	0	1	0	0	1	0	0	1	0	18	0.944461
12	0	0	1	0	1	0	1	0	0	0	0	1	17	0.753772
13	0	0	1	0	0	1	0	1	0	1	0	0	4	-1.65823
14	0	0	1	0	0	1	1	0	0	0	1	0	2	-2.44235
15	0	0	0	1	1	0	0	0	0	0	0	1	23	2.442347
16	0	0	0	1	1	0	1	0	0	0	1	0	3	-1.99243
17	0	0	0	1	0	1	0	0	1	0	1	0	5	-1.38629
18	0	1	0	1	0	1	0	0	1	0	0	1	19	1.15268
19	1	0	0	0	0	1	0	1	0	1	0	0	13	0.080043
20	0	1	0	0	1	0	0	0	1	1	0	0	15	0.405465
21	0	1	0	0	0	1	0	1	0	0	0	1	9	-0.57536
22	0	0	1	0	0	1	0	1	0	0	1	0	7	-0.94446
23	0	0	0	1	1	0	0	1	0	0	1	0	14	0.241162
24	0	0	0	1	0	1	0	1	0	1	0	0	1	-3.17805

Linear dependency exists in the binary coding problem. In order to account for it, the binary matrix is modified by choosing a reference level for which the part-worth utilites are based on. Effects coding is used to modify the matrix, which sets the reference level at '-1'. The reference attributes are chosen are DS of ~0 cm, GS of Easy to manufacture, MM of Poor and PA of Poor. The modified table is displayed in Table 7.7.

Table 7.7: Dummy-Variable Binary Representation

Comb.	***DS(~cm)***				***GS***		***MM***			***PA***			***Rank***	***y***
	0	10	20	30	E	H	P	M	G	P	M	G		
1	-1	0	0	0	-1	0	-1	1	0	-1	0	0	18	1.658228
2	-1	0	0	0	-1	0	-1	0	0	-1	1	0	14	1.992430
3	-1	0	0	0	-1	0	-1	0	1	-1	0	1	4	3.178053
4	-1	0	0	0	-1	1	-1	0	1	-1	1	0	15	0.575364
5	-1	0	0	0	-1	1	-1	1	0	-1	0	1	13	-0.08004
6	-1	1	0	0	-1	0	-1	0	0	-1	0	0	10	-0.24116
7	-1	1	0	0	-1	0	-1	1	0	-1	1	0	16	1.386294
8	-1	1	0	0	-1	1	-1	0	1	-1	0	0	8	-1.15268
9	-1	1	0	0	-1	1	-1	0	1	-1	1	0	3	-0.75377
10	-1	0	1	0	-1	0	-1	0	0	-1	0	0	17	-0.40546
11	-1	0	1	0	-1	0	-1	1	0	-1	1	0	9	0.944461
12	-1	0	1	0	-1	0	-1	0	0	-1	0	1	7	0.753772
13	-1	0	1	0	-1	1	-1	1	0	-1	0	0	11	-1.65823
14	-1	0	1	0	-1	1	-1	0	0	-1	1	0	5	-2.44235
15	-1	0	0	1	-1	0	-1	0	0	-1	0	1	1	2.442347
16	-1	0	0	1	-1	0	-1	0	0	-1	1	0	12	-1.99243
17	-1	0	0	1	-1	1	-1	0	1	-1	1	0	6	-1.38629
18	-1	1	0	1	-1	1	-1	0	1	-1	0	1	7	1.15268
19	-1	0	0	0	-1	1	-1	1	0	-1	0	0	11	0.080043
20	-1	1	0	0	-1	0	-1	0	1	-1	0	0	5	0.405465
21	-1	1	0	0	-1	1	-1	1	0	-1	0	1	1	-0.57536
22	-1	0	1	0	-1	1	-1	1	0	-1	1	0	12	-0.94446
23	-1	0	0	1	-1	0	-1	1	0	-1	1	0	6	0.241162
24	-1	0	0	1	-1	1	-1	1	0	-1	0	0	2	-3.17805

A Regression Analysis is conducted using ANOVA Regression tool in Excel as described by Equation (4.3). In order to calculate the part-wroth utilities for the levels of each attribute, the regression model is solved with the variables in the binary matrix being the independent variables and the logit recoded rankings(*y*) being the dependent variables. The results of the Regression are displayed in Figure 7.3.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.92628302
R Square	0.85800022
Adjusted R Square	0.51560034
Standard Error	0.72882007
Observations	24

ANOVA

	df	SS	MS	F	Significance F
Regression	12	48.14283376	4.011903	11.32925	0.000159859
Residual	15	7.967680438	0.531179		
Total	27	56.1105142			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	0.56150399	0.478565857	1.173306	0.258968	-0.45853498	1.581543	-0.458535	1.581542966
X Variable 1	0	0	65535	#NUM!	0	0	0	0
X Variable 2	-1.2820488	0.433065425	-2.9604	0.009726	-2.20510591	-0.358992	-2.2051059	-0.35899171
X Variable 3	-1.0874895	0.451670123	-2.40771	0.029379	-2.05020153	-0.124777	-2.0502015	-0.12477738
X Variable 4	-1.9060851	0.428827754	-4.44487	0.000473	-2.82010984	-0.99206	-2.8201098	-0.9920604
X Variable 5	0	0	65535	#NUM!	0	0	0	0
X Variable 6	-2.2033707	0.327296553	-6.73203	6.73E-06	-2.90098683	-1.505755	-2.9009868	-1.50575466
X Variable 7	0	0	65535	#NUM!	0	0	0	0
X Variable 8	1.02133664	0.415410486	2.45862	0.026587	0.135910149	1.9067631	0.13591015	1.906763123
X Variable 9	1.83573113	0.483387575	3.797638	0.001752	0.805414906	2.8660474	0.80541491	2.866047351
X Variable 10	0	0	65535	#NUM!	0	0	0	0
X Variable 11	0.49745026	0.356270807	1.39627	0.182955	-0.26192299	1.2568235	-0.261923	1.256823504
X Variable 12	1.45739591	0.414499482	3.516038	0.003119	0.573911179	2.3408806	0.57391118	2.340880637

Figure 7.3 Regression Statistics for ANOVA Regression for the Clutch Problem

The intercept values represent the corresponding part-worth utilities for the levels of each attributes. The results indicate a good fit to the regression model formed by the combinations and their rankings. The part-worths for the reference values are seen to be zero and the values for the other intercepts correspond to the preferences in reference to these levels. The part-worth utilities are scaled to be positive by adding the minimum utility from a specified attribute to all other levels for that attribute.

Once the part-worth utilities are obtained for each level of each attribute, the importance of each attribute is calculated. The importance value is the difference an attribute contributes to the total utility of a product. As shown in Table 7.8, the difference is the range in the utilities for each attribute. Percentage values, that add up to 100%, are calculated from the ranges. For the given application, DS has an importance of 25.75%, GS has an importance of 29.77%, MM of 24.79% and PA of 19.69%.

Table 7.8: Calculation of Attribute Importance

Attributes	*Levels*	*Part-Worth Utilities*	*Range*	*Attribute Importance*
DS	~0 cm ~10 cm ~20 cm ~30 cm	1.9061 0.62404 0.8186 0	1.9061-0=1.9061	(1.9061/7.4026)*100=**25.75%**
GS	Easy(E) Hard(H)	2.2033 0	2.2033-0=2.2033	(2.2033/7.4026)*100=**29.77%**
MM	Poor Medium Good	0 1.0213 1.8357	1.8357-0=1.8357	(1.8356/7.4026)*100=**24.79%**
PA	Poor Medium Good	0 0.4975 1.4574	1.4574-0=1.4574	(1.4574/7.4026)*100=**19.69%**
Total			7.4026	

7.1.3 Tolerance allocation Using FCE

The machinability values are determined for the parts using the FCE method in Chapter 3.

Table 7.9: Machinability Values for Parameters

Parameter	Machinability ζ
T_f	0.6127
T_p	0.6498
T_c	0.3458
T_{0a}	0.7309
L_{ha}	0.5992
R_{cr}	0.3751
R_{da}	0.3991

In an ideal friction clutch with no tolerances and no gaps/interferences, the equations in the horizontal direction for the dimensions, given in Figure 7.2, is:

$$T_{0a} - T_f - T_c - T_p = 0 \tag{7.1}$$

Once the tolerances for each dimension are introduced in the process, Equation (7.1) changes to

$$(T_{0a} + Tol_{T_{0a}}) - \left(T_f + Tol_{T_f}\right) - \left(T_c + Tol_{T_c}\right) - (T_p + Tol_{T_p}) = Tol_{L1} \tag{7.2}$$

where Tol_{L1} is the tolerance of location L_1 in the assembly in Figure 7.2. Tol_i is the tolerance of assembly part i. After combining Equations (7.1) and (7.2) one gets:

$$Tol_{T_{0a}} - Tol_{T_f} - Tol_{T_c} - Tol_{T_p} = Tol_{L1} \tag{7.3}$$

Using a similar process, the tolerance for gap L_2, Tol_{L2}, is calculated as:

$$\frac{Tol_{L_{ha}}}{2} - Tol_{R_{da}} - Tol_{R_{cr}} = Tol_{L2} \tag{7.4}$$

The dimensions L_{ha}, R_{da} and R_{cr} are shown in Figure 7.2.

Equations (7.3) and (7.4) are the assembly function equations described by Equation (3.14) in Chapter 3. The coefficients of the tolerances in these equations are the values of the sensitivity coefficient, ξ, in the equation. The comprehensive factor, Ψ, is calculated using Equation (3.15). The values are listed in Table 7.10.

Table 7.10: Comprehensive Factor Values for Parameters

Parameter	Sensitivity coefficient ξ	Comprehensive factor Ψ
T_f	-1	0.6127
T_p	-1	0.6498
T_c	-1	0.3458
T_{0a}	1	0.7309
L_{ha}	½	2.3968
R_{cr}	-1	0.3751
R_{da}	-1	0.3991

The total machining cost is obtained using Equation 3.16.

$$C_m = C_0 + \frac{0.6127}{Tol_{T_f}} + \frac{0.6498}{Tol_{T_p}} + \frac{0.3458}{Tol_{T_f}} + \frac{0.7309}{Tol_{T_{0a}}} + \frac{2.3968}{Tol_{L_{ha}}} + \frac{0.3751}{Tol_{R_{cr}}} + \frac{0.3991}{Tol_{R_{da}}} \quad (7.5)$$

Where C_m is the total cost of machining, and C_0 is the initial setup costs. C_0 is \$20 for this application.

7.1.4 Determining Costs due to Performance Variation

The performance of a clutch is determined by its clutch torque capacity. The formula for clutch torque capacity[63], *CTC*, is given by:

$$CTC = P\mu N R_g \tag{7.6}$$

where P is the clamping force provided by the clutch housing, μ is the coefficient of friction, N is the number of surfaces and R_g is the radius of gyration.

The clamping force, P, is provided by the manufacturer. It is assumed to be 450 gms for the housing used. N is 2 for a single disk clutch. μ is 0.53 for the friction material used. The radius of gyration, R_g, is calculated using:

$$R_g = \sqrt{R_d^2 - R_i^2} \tag{7.7}$$

where R_d is the outer clutch radius, R_i is inner clutch radius. The inner clutch radius is assumed to be constant since it is not part of the tolerance equation. R_i is 8 cm for the application. The clutch torque capacity for the current configuration that does not take tolerances into account is 10.73 kg.m. A CTC that is too high will cause damage to the clutch, while too low a CTC will not provide enough torque for clutch functioning. Thus, a 'nominal is best' Taguchi loss function is used. When tolerances are included in the equation, the clutch torque capacity, CTC, now becomes:

$$CTC = P\mu N(\left(R_d + Tol_{R_{da}} + Tol_{R_{cr}}\right)^2 - R_i^2) \tag{7.8}$$

The costs incurred due to variation in performance, C_L, are given by:

$$C_L = k(CTC - 10.73)^2 \tag{7.9}$$

where k is the clutch constant. The value of k is determined by the cost required to replace a clutch that deviates from its expected performance by a certain amount. The cost of fixing a clutch that deviates from its targeted performance by 5 kg.m is \$12.5.

$$12.5 = k(5)^2$$

$$k = 0.5$$

Equation (7.9) now becomes:

$$C_L = 0.5(CTC - 10.73)^2 \quad (7.10)$$

7.1.5 Optimization of Cost

The final equal to be used for optimization is:

$$C = C_M + C_L \quad (7.11)$$

where C_M is described by Equation (7.5) and C_L by (7.10). The constraints used for optimization are shown in Table 7.11. The values included are the absolute values for the bounds.

Table 7.11: Upper and Lower Bounds for Tolerances

Parameter	Lower bound (cm)	Upper bound (cm)
Tol_{Tf}	0.1	0.4
Tol_{Tp}	0.1	0.6
Tol_{Tc}	0.01	0.1
Tol_{T0a}	0.1	1
Tol_{Lha}	0.1	2.6
Tol_{Rcr}	0.1	0.2
Tol_{Rda}	0.1	0.9
Tol_{L1}	0	0.3
Tol_{L2}	0	0.2

The optimization is performed, minimizing the total cost *C*, under the constraints in Table 7.11. The obtained outputs for the tolerances and the performance are included in Table 7.12.

Table 7.12: Optimal Tolerance Values

Parameter	Tolerance values (cm)
Tol_{Tf}	0.4
Tol_{Tp}	0.6
Tol_{Tc}	0.1
Tol_{T0a}	1
Tol_{Lha}	0.9757
Tol_{Rcr}	0.1461
Tol_{Rda}	0.1417
CTC	13.32 kg.m

The torque capacity of the optimized clutch is 13.32 kg.m, which is a deviation of 2.41% from the expected torque capacity.

There is an interference of 0.1 cm at location L_1 and a gap of 0.2 cm at L_2, which is within the allowable constraints for proper functioning of the clutch.

7.1.6 Validation

7.1.6.1 Cross-Validation

The 3-fold cross-validation method, described in Section 6.3, is used to confirm the utility of CA in the application.

The cross-validation method is first applied to the proposed framework that utilizes conjoint analysis. The 24 combinations in Table 7.4 are divided into 3 test sets of 8 each. The remainder in the complete set after each test set is removed is the training set. These are:

Training set 1: Combinations 2, 6, 9, 11, 14, 17, 20 and 22 are removed

Training set 2: Combinations 1, 3, 5, 10, 13, 16, 19, and 21 are removed

Training set 3: Combinations 4, 7, 8, 12, 15, 18, 23, and 24 are removed

The tolerance allocation framework in Chapter 6 is applied to each training set and the complete set. The RMS error for each training set is calculated using Equation (6.5) and the total error is obtained using Equation (6.6).

The same procedure is applied to the proposed framework that does not utilize conjoint analysis. The importance vector is determined by the percentage preferences of a 100 experts. Out of a 100 experts, 30% voted for factor DS, 30% for GS, 30% for MM and 10% for PA. The 100 experts are divided into 3 test sets: two of them with 33 experts and one with 34 experts.

Training set 1: 15 of those who voted for DS, 15 for GS and 3 for PA are removed

Training set 2: 30 of those who voted for MM, 3 for GS are removed

Training set 3: 15 voted for DS, 12 for GS, 7 for PA are removed

The tolerance allocation framework in Chapter 6 is applied to each training set and the complete set. The RMS error for each training set is calculated using Equation (6.5) and the total error is obtained using Equation (6.6). The cross-validation results are shown in Table 7.13.

Table 7.13: Cross-Validation of Tolerance Allocation Framework

	$RMS_{e,total}$
Tolerance Allocation with CA	0.0116
Tolerance allocations without CA	0.0197

There is a decrease of 41.09% in the total RMS error value on utilization of CA framework in FCE process. This demonstrates that the need for conjoint analysis in the framework.

7.1.6.2 Validation of Taguchi's Loss function

The propose framework is applied with and without use of Taguchi's loss function. The deviation from the expected clutch torque capacity is noted and displayed in Table 7.14.

Table 7.14: Deviation in Performance with and without Taguchi's Quality Loss Function

	CTC(kg.m)	*Percentage Deviation*	*Quality Loss Costs($)*
Tolerance Allocation with Taguchi	13.3211	24.1%	$3.368
Tolerance allocations without Taguchi	21.0643	96.4%	$53.43

The results indicate that there is a significant increase in robustness achieved due to use of Taguchi's quality loss function. The deviation in CTC is four times higher if Taguchi's quality loss function is not utilized. If the torque capacity is too high, transmission life is shortened and there are design drawbacks that occur due to higher maintenance costs, pedal effort, wear rate, noise, chatter etc [63].

7.2 O-ring Seal in an Accumulator

A popular use of hydraulic power in the aerospace industry is with piston actuated accumulators. An accumulator is the hydraulic equivalent to an electrical capacitor; it stores potential energy in a system and may release it as needed. Accumulators provide the consistent pressure needed in a hydraulic system during pressure transients when large actuators are in use (such as flight controls and landing gear systems). This is done by separating the hydraulic fluid with a bladder or piston, where one side would have a spring or certain gas at a pressurized amount, and the other an incompressible fluid. This guarantees that a 'pre-charge' will always be applied to the hydraulic fluid. Because the pre-charge is compressible, accumulators also absorb hydraulic pressure spikes, and can cushion load.

For optimum performance of an accumulator, everything from thermal affects to seal-fluid interaction is scrutinized, especially in industries that have experience with extreme environments (such as aerospace). One of the most common problems with accumulators is leakage. In any hydraulic component, seals are employed to reduce leakage in static or dynamic applications, and are designed to 'sit' in a groove of a machined part (commonly referred to as the seal gland). Figure 7.4 shows the sub-components of an accumulator that would require seals. Because different hydraulic fluids are available, seal compatibility is critical, especially at the temperature extremes. Incompatibility could result in improper swell rates and/or chemical breakdown of the seal. O-ring seals are generally used in aircrafts accumulator.

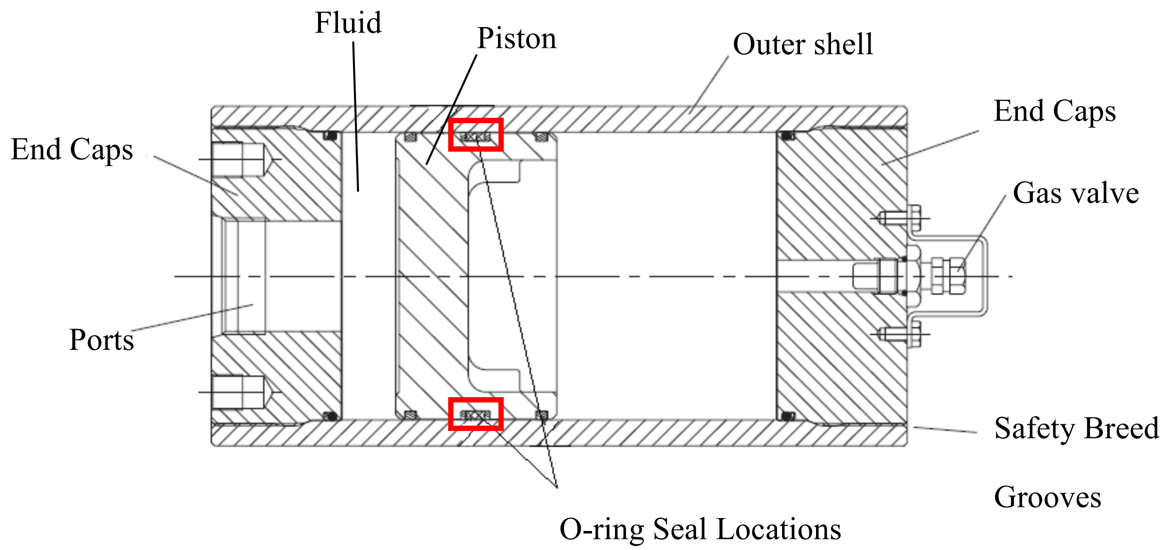


Figure 7.4 Cross-section of an Accumulator with O-ring Seal Locations[64]

O-ring seal

An O-ring seal, shown in Figure 7.5, is used to prevent loss of fluid or gas from the accumulator. It consists of an O-ring made of a Polytetrafluoroethylene (PTFE), a thermoplastic and a supporting gland. The O-ring is a circular cross section molded from rubber. The gland is the housing that enables the seal to function correctly. The volume of the gland is dependent upon the manufacturing tolerances of the parts and the tolerance of the seal itself [64].

Tolerance allocation is of vital importance in the functioning of the seal assembly. Improper tolerance allocation can result in malfunctioning of the seal which is a detriment to the safety of the aircraft passenger. The gap or interference at locations L_1 or L_2 is the overall tolerance of the seal assembly. If there is a significant gap at L_1 or L_2, then the O-ring is undersized for the gland. As a result, the O-ring will not swell to a point that it will be able to seal. If there is a significant interference at L_1 or L_2, then the O-ring will swell and try to overfill the gland. This results in physical damage to the seal. Several repeated thermal cycles of expansion and contraction would reduce the life of the seal (or similar to relaxing and squeezing the balloon). In aerospace this would be due to the number of flights, and larger temperature changes exacerbate the problem.

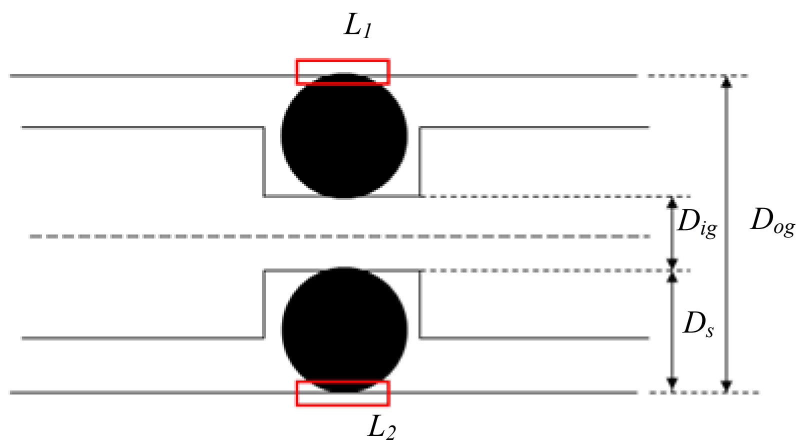


Figure 7.5: Accumulator O-ring Seal Dimensions

The dimensioning used for tolerance allocation is shown in Figure 7.5 and the corresponding variables are listed in Table 7.15. The grades of the fuzzy factors are listed in Table 7.16.

Table 7.15: Variable Descriptions and Dimensions for the O-ring Seal Problem

Variable	**Description**	**Value (cm)**
D_s	Seal diameter	2
D_{ig}	Inner Gland Diameter	3
D_{og}	Outer Gland Diameter	7

Table 7.16: Grade Divisions of Fuzzy factors for the O-ring Seal Problem

	Grade 1	**Grade 2**	**Grade 3**	**Grade 4**
U_1 (DS)	~1 cm	~3 cm	~5 cm	~7 cm
U_2 (GS)	Easy to manufacture	Hard to manufacture	-	-
U_3(MM)	Poor	Medium	Good	-
U_4 (PA)	Poor	Medium	Good	-

7.2.1 Membership grade elicitation

To conduct the FCE process, the fuzzy subsets of the four factors, DS, GS, MM and PA (with fuzzy subsets U_1 to U_4) are determined first. The first factor, Dimension Size (DS), is the dimension of the part. The fuzzy subset for DS (U_1) is determined based on the value in Table 7.14 and listed in Table A.5 in the Appendix.

Membership grade for GS, MM and PA are determined using the rank ordering method described in Chapter 3.2 Step 1. 100 experts are asked to compare two grades of each factor as to which one of them is better suited to describe the fuzzy factor for the part. This comparison is done for all combinations of grade pairs. A percentage preference is established for each grade which is the membership degree value of the grade for the assembly part.

For the fuzzy factor GS, the fuzzy subset (U_2) is listed in Table A.6 in the Appendix. Inner gland is hardest to machine since its shape is more irregular than that of the seal or outer gland.

For the fuzzy factor MM, the fuzzy subset (U_3) is listed in Table A.7 as the Appendix. The experts determine the MM value based on the material malleability. A component, whose material has higher malleability, has higher MM. PTFE is the most malleable, while steel is the least. The list of materials is shown in Table 7.17.

Table 7.17: Materials used for the O-ring Seal

Assembly part	Material
Seal	PTFE
Inner gland	Aluminum
Outer gland	Steel

For the fuzzy factor PA, fuzzy subset (U_4) is listed as Table A.8 in the Appendix. The experts rank order the subsets for each part based on how often the part is in contact with other parts of the system. The more contact that occurs for a part while compressing the seal, the higher the required PA is for proper functioning of the system. The seal is assumed to have a higher PA value since it has a higher contact area during compression than the glands.

Once the fuzzy subsets are determined, the next step is to obtain the 1st order FCE matrix, shown in Equation 3.4. The matrices for each of the factors are evaluated by experts as described for the clutch application in Section 7.1.1. The four matrices R_{DS}, R_{GS}, R_{MM} and R_{PA} are the 1st order FCE matrices for the factors GS, MM and PA respectively.

$$R_{DS}=\begin{bmatrix} 0.8 & 1 & 0.8 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0.8 & 1 & 0.8 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0.7 & 1 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.8 & 1 \end{bmatrix}$$

$$R_{GS}=\begin{bmatrix} 0 & 0.5 & 0.8 & 1 & 0.7 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 1 & 0.7 \end{bmatrix}$$

$$R_{MM}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 & 1 & 0.8 \\ 0 & 0 & 0 & 0 & 0.7 & 1 & 0.7 & 0.05 & 0 & 0 \\ 1 & 0.3 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and $$R_{PA}=\begin{bmatrix} 0.8 & 1 & 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0.7 & 1 & 0.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0.8 & 1 \end{bmatrix}$$

7.2.2 Determining Importance Factors Using Conjoint Analysis

The CA method, described in Chapter 4, is applied to evaluate the importance factor, *I*. The four grades, DS, GS, MM and PA the four attributes and they have four, two, three and three levels respectively. Thus, there are 4*2*3*3=72 combinations. To make ranking easier for the experts, a 1/3 fractional factorial is taken, resulting in a total of 24 combinations [45]. A fractional factorial design will take an adequate fraction of combinations, with as little an effect on the final combination as possible. The set of 24 combinations that are used for the Conjoint Analysis are shown in Table 7.18.

Table 7.18: Fractional Factorial Design for O-ring Seal Application

Combination	***DS***	***GS***	***MM***	***PA***
1	~1 cm	Easy To Manufacture	Medium	Poor
2	~1 cm	Easy To Manufacture	Poor	Medium
3	~1 cm	Easy To Manufacture	Hard	Hard
4	~1 cm	Hard to manufacture	Hard	Medium
5	~1 cm	Hard to manufacture	Medium	Hard
6	~1 cm	Hard To Manufacture	Medium	Poor
7	~3 cm	Easy To Manufacture	Poor	Poor
8	~3 cm	Easy To Manufacture	Medium	Medium
9	~3 cm	Hard to manufacture	Hard	Poor
10	~3 cm	Hard to manufacture	Hard	Medium
11	~3 cm	Easy To Manufacture	Hard	Poor
12	~3 cm	Hard To Manufacture	Medium	Hard
13	~5 cm	Easy to manufacture	Poor	Poor
14	~5 cm	Easy to manufacture	Medium	Medium
15	~5 cm	Easy To Manufacture	Poor	Hard
16	~5 cm	Hard To Manufacture	Medium	Poor
17	~5 cm	Hard to manufacture	Poor	Poor
18	~5 cm	Hard to manufacture	Medium	Medium
19	~7 cm	Easy to manufacture	Hard	Hard
20	~7 cm	Easy To Manufacture	Poor	Medium
21	~7 cm	Hard to manufacture	Hard	Medium
22	~7 cm	Hard to manufacture	Hard	Hard
23	~7 cm	Easy To Manufacture	Medium	Medium
24	~7 cm	Hard to manufacture	Poor	Poor

The experts' preferences for the above combinations are obtained through customer surveys, computer programs or elicitations from designers. The subjective data elicited are based on the attributes and preferences of the responded. This is shown in Table 7.19.

Table 7.19: Expert Rating of Attribute Combinations

Comb.	***DS***	***GS***	***MM***	***PA***	***Rank***
1	~1 cm	Easy To Manufacture(E)	Medium(M)	Poor(P)	8
2	~1 cm	Easy To Manufacture(E)	Poor(P)	Medium(M)	9
3	~1 cm	Easy To Manufacture(E)	Hard(H)	Hard(H)	24
4	~1 cm	Hard to manufacture(H)	Hard(H)	Medium(M)	21
5	~1 cm	Hard to manufacture(H)	Medium(M)	Hard(H)	22
6	~1 cm	Hard To Manufacture(H)	Medium(M)	Poor(P)	5
7	~3 cm	Easy To Manufacture(E)	Poor(P)	Poor(P)	4
8	~3 cm	Easy To Manufacture(E)	Medium(M)	Medium(M)	10
9	~3 cm	Hard to manufacture(H)	Hard(H)	Poor(P)	7
10	~3 cm	Hard to manufacture(H)	Hard(H)	Medium(M)	19
11	~3 cm	Easy To Manufacture(E)	Hard(H)	Poor(P)	13
12	~3 cm	Hard To Manufacture	Medium(M)	Hard(H)	20
13	~5 cm	Easy to manufacture(E)	Poor(P)	Poor(P)	6
14	~5 cm	Easy to manufacture(E)	Medium(M)	Medium(M)	14
15	~5 cm	Easy To Manufacture(E)	Poor(P)	Hard(H)	18
16	~5 cm	Hard To Manufacture(H)	Medium(M)	Poor(P)	3
17	~5 cm	Hard to manufacture(H)	Poor(P)	Poor(P)	2
18	~5 cm	Hard to manufacture(H)	Medium(M)	Medium(M)	15
19	~7 cm	Easy to manufacture(E)	Hard(H)	Hard(H)	23
20	~7 cm	Easy To Manufacture(E)	Poor(P)	Medium(M)	17
21	~7 cm	Hard to manufacture(H)	Hard(H)	Medium(M)	12
22	~7 cm	Hard to manufacture(H)	Hard(H)	Hard(H)	16
23	~7 cm	Easy To Manufacture(E)	Medium(M)	Medium(M)	11
24	~7 cm	Hard to manufacture(H)	Poor(P)	Poor(P)	1

The Dummy Variable Regression table is shown in Table 7.20. The y-values are calculated using Logit coding in Equation (4.4-4.5).

Table 7.20: Dummy-Variable Binary Representation

Comb.	***DS(~cm)***				***GS***		***MM***			***PA***			***Rank***	***y***
	0	10	20	30	E	H	P	M	G	P	M	G		
1	1	0	0	0	1	0	0	1	0	1	0	0	8	-0.7538
2	1	0	0	0	1	0	1	0	0	0	1	0	9	-0.5754
3	1	0	0	0	1	0	0	0	1	0	0	1	24	3.17805
4	1	0	0	0	0	1	0	0	1	0	1	0	21	1.65823
5	1	0	0	0	0	1	0	1	0	0	0	1	22	1.99243
6	1	0	0	0	0	1	0	1	0	1	0	0	5	-1.3863
7	0	1	0	0	1	0	1	0	0	1	0	0	4	-1.6582
8	0	1	0	0	1	0	0	1	0	0	1	0	10	-0.4055
9	0	1	0	0	0	1	0	0	1	1	0	0	7	-0.9445
10	0	1	0	0	0	1	0	0	1	0	1	0	19	1.15268
11	0	1	0	0	1	0	0	0	1	1	0	0	13	0.08004
12	0	1	0	0	0	1	0	1	0	0	0	1	20	1.38629
13	0	0	1	0	1	0	1	0	0	1	0	0	6	-1.1527
14	0	0	1	0	1	0	0	1	0	0	1	0	14	0.24116
15	0	0	1	0	1	0	1	0	0	0	0	1	18	0.94446
16	0	0	1	0	0	1	0	1	0	1	0	0	3	-1.9924
17	0	0	1	0	0	1	1	0	0	1	0	0	2	-2.4423
18	0	0	1	0	0	1	0	1	0	0	1	0	15	0.40547
19	0	0	0	1	1	0	0	0	1	0	0	1	23	2.44235
20	0	0	0	1	1	0	1	0	0	0	1	0	17	0.75377
21	0	0	0	1	0	1	0	0	1	0	1	0	12	-0.08
22	0	0	0	1	0	1	0	0	1	0	0	1	16	0.57536
23	0	0	0	1	1	0	0	1	0	0	1	0	11	-0.2412
24	0	0	0	1	0	1	1	0	0	1	0	0	1	-3.1781

Linear dependency exists in the binary coding problem. In order to account for it, the binary matrix is modified by choosing a reference level for the part-worth utilities are based on. Effects coding is used to modify the matrix by setting a reference level at '-1'. The reference attributes are chosen are DS of ~0 cm, GS of Easy to manufacture, MM of Poor and PA of Poor. The modified table is displayed in Table 7.21.

Table 7.21: Dummy-Variable Binary Representation

Comb.	***DS(~cm)***				***GS***		***MM***			***PA***			***Rank***	***Y***
	0	10	20	30	E	H	P	M	G	P	M	G		
1	-1	0	0	0	-1	0	-1	1	0	-1	0	0	8	-0.7538
2	-1	0	0	0	-1	0	-1	0	0	-1	1	0	9	-0.5754
3	-1	0	0	0	-1	0	-1	0	1	-1	0	1	24	3.17805
4	-1	0	0	0	-1	1	-1	0	1	-1	1	0	21	1.65823
5	-1	0	0	0	-1	1	-1	1	0	-1	0	1	22	1.99243
6	-1	0	0	0	-1	1	-1	1	0	-1	0	0	5	-1.3863
7	-1	1	0	0	-1	0	-1	0	0	-1	0	0	4	-1.6582
8	-1	1	0	0	-1	0	-1	1	0	-1	1	0	10	-0.4055
9	-1	1	0	0	-1	1	-1	0	1	-1	0	0	7	-0.9445
10	-1	1	0	0	-1	1	-1	0	1	-1	1	0	19	1.15268
11	-1	1	0	0	-1	0	-1	0	1	-1	0	0	13	0.08004
12	-1	1	0	0	-1	1	-1	1	0	-1	0	1	20	1.38629
13	-1	0	1	0	-1	0	-1	0	0	-1	0	0	6	-1.1527
14	-1	0	1	0	-1	0	-1	1	0	-1	1	0	14	0.24116
15	-1	0	1	0	-1	0	-1	0	0	-1	0	1	18	0.94446
16	-1	0	1	0	-1	1	-1	1	0	-1	0	0	3	-1.9924
17	-1	0	1	0	-1	1	-1	0	0	-1	0	0	2	-2.4423
18	-1	0	1	0	-1	1	-1	1	0	-1	1	0	15	0.40547
19	-1	0	0	1	-1	0	-1	0	1	-1	0	1	23	2.44235
20	-1	0	0	1	-1	0	-1	0	0	-1	1	0	17	0.75377
21	-1	0	0	1	-1	1	-1	0	1	-1	1	0	12	-0.08
22	-1	0	0	1	-1	1	-1	0	1	-1	0	1	16	0.57536
23	-1	0	0	1	-1	0	-1	1	0	-1	1	0	11	-0.2412
24	-1	0	0	1	-1	1	-1	0	0	-1	0	0	1	-3.1781

A Regression Analysis is conducted using ANOVA Regression tool in Excel as described by Equation (4.3). In order to calculate the part-wroth utilities for the levels of each attribute, the regression model is solved with the variables in the binary matrix being the independent variables and the logit recoded rankings(y) being the dependent variables. The results of the Regression are displayed in Figure 7.6.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.94354395
R Square	0.89027519
Adjusted R Square	0.56508863
Standard Error	0.64066191
Observations	24

ANOVA

	df	*SS*	*MS*	*F*	*ignificance F*
Regression	12	49.95379896	4.162817	15.21321	3.807E-05
Residual	15	6.156715239	0.410448		
Total	27	56.1105142			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-1.24183693	0.393173284	-3.1585	0.006493	-2.079866	-0.40380791	-2.0798659	-0.40380791
X Variable 1	0	0	65535	#NUM!	0	0	0	0
X Variable 2	-0.43801366	0.380381	-1.15151	0.267546	-1.248777	0.37274924	-1.2487766	0.372749242
X Variable 3	-0.37924885	0.391620057	-0.96841	0.348196	-1.213967	0.45546954	-1.2139672	0.455469539
X Variable 4	-1.01123209	0.388262358	-2.60451	0.019919	-1.838794	-0.18367047	-1.8387937	-0.18367047
X Variable 5	0	0	65535	#NUM!	0	0	0	0
X Variable 6	-0.61998501	0.279241892	-2.22024	0.04223	-1.215175	-0.02479501	-1.215175	-0.02479501
X Variable 7	0	0	65535	#NUM!	0	0	0	0
X Variable 8	0.45582935	0.355629051	1.281755	0.219387	-0.302176	1.21383472	-0.302176	1.213834724
X Variable 9	1.47188265	0.397747141	3.700549	0.002137	0.6241047	2.31966061	0.62410469	2.319660609
X Variable 10	0	0	65535	#NUM!	0	0	0	0
X Variable 11	1.66611121	0.32558128	5.117343	0.000126	0.9721511	2.36007128	0.97215114	2.360071277
X Variable 12	2.8903913	0.367496902	7.865077	1.06E-06	2.1070902	3.6736924	2.1070902	3.673692398

Figure 7.6: Regression statistics for ANOVA Regression for O-ring Seal Problem

The intercept values represent the corresponding part-worth utilities for the levels of each attributes. The results indicate a good fit to the regression model formed by the combinations and their rankings. The part-worths for the reference values are seen to be zero and the values for the other intercepts correspond to the preferences in reference to these levels. The part-worth utilities are scaled to be positive by adding the minimum utility from a specified attribute to all other levels for that attribute.

Once the part-worth utilities are obtained for each level of each attribute, the importance of each attribute is calculated. The importance value is the difference an attribute contributes to the total utility of a product. As shown in Table 7.22, the difference is the range in the utilities for each attribute. Percentage values, that add up to 100%, are calculated from the ranges. For the given application, DS has an importance of 16.87%, GS has an importance of 10.34%, MM of 24.56% and PA of 48.22%. The factors DS and GS have low importance values as compared to the friction clutch. This reflects the fact there is little variability in the dimension sizes and shapes of the O-ring seal as compared to the clutch.

Table 7.22: Calculation of Attribute Importance

Attribute	*Levels*	*Part-Worth Utilities*	*Range*	*Attribute Importance*
DS	~1 cm ~3 cm ~5 cm ~7 cm	1.0112 0.5732 0.6320 0	1.0112-0=1.0112	(1.0112/5.9935)*100=**16.87%**
GS	Easy(E) Hard(H)	0.6199 0	0.6199-0=0.6199	(0.6199/5.9935)*100=**10.34%**
MM	Poor Medium Good	0 0.4558 1.4719	1.4719-0=1.4719	(1.4719/5.9935)*100=**24.56%**
PA	Poor Medium Good	0 1.6661 2.8904	2.8904-0=2.8904	(2.8904/5.9935)*100=**48.22%**
Total			5.9935	

7.2.3 Tolerance Allocation using FCE

The machinability values are determined for the parts using the FCE method in Chapter 3 and listed in Table 7.23.

Table 7.23: Machinability values for Parameters

Parameter	Machinability ζ
D_s	0.6289
D_{ig}	0.6160
D_{og}	0.6343

In an ideal O-ring seal that does not have dimensional uncertainties, the equation for the gaps at Location L_1 and L_2, in Figure 7.5, is:

$$D_{og} - D_{ig} - 2D_s = 0 \tag{7.12}$$

On including tolerances, Equation (7.11) becomes:

$$(D_{og} + Tol_{D_{og}}) - \left(D_{ig} + Tol_{D_{ig}}\right) - 2\left(D_s + Tol_{D_s}\right) = 2Tol_{L_1} = 2Tol_{L_2} \tag{7.13}$$

On combining Equations (7.11) and (7.12) one obtains:

$$\frac{Tol_{D_{og}} - Tol_{D_{ig}} - 2Tol_{D_s}}{2} = Tol_{L1} = Tol_{L2} \tag{7.14}$$

where Tol_{L1} and Tol_{L2} are the gaps at locations L_1 and L_2 respectively. The dimensions D_{og}, D_{ig} and D_s are shown in Figure 7.5.

Equation (7.13) is the assembly function equation described by Equation (3.14) in Chapter 3. The coefficients of the tolerances in these equations are the values of the sensitivity coefficient, ξ, in the equation. The comprehensive factor, Ψ, is calculated using Equation (3.15). The values are listed in Table 7.24.

Table 7.24: Comprehensive Factor Values for Parameters

Parameter	Sensitivity coefficient ξ	Comprehensive factor Ψ
D_s	-1	0.6289
D_{ig}	-1/2	2.4640
D_{og}	-1/2	2.5372

The total machining cost is obtained using Equation (3.16).

$$C_m = C_0 + \frac{0.6289}{Tol_{D_s}} + \frac{2.4640}{Tol_{D_{ig}}} + \frac{2.5372}{Tol_{D_{og}}} \quad (7.15)$$

where C_m is the total cost of machining, and C_0 is the initial setup costs. C_0 is \$5 for this application

7.2.4 Determining Costs due to Performance Variation

The dimensional parameters in the O-ring seal do not have a direct impact on the performance of the accumulator. Any effects on the performance due to leakage of seal are prevented by ensuring that the squeeze is below 18% for the seal. If the squeeze is above 18%, the seal will overfill the gland, damaging the seal. If the squeeze is less than -18%, the seal does not swell to a point that it will seal. Both cases can allow external gases to enter the accumulator and cause damages that affect performance.

The squeeze, Sq, is the tolerance of the locations as a fraction of the seal diameter. This constraint is indicated by Equation (7.15) and (7.16).

$$Sq = \frac{Tol_{L_1}}{D_s} = \frac{Tol_{L_2}}{D_s} \quad (7.16)$$

$$|Sq| < 0.18 \quad (7.17)$$

This ensures that there is no cost incurred due to variation in performance, as shown by Equation (7.17).

$$C_L = 0 \tag{7.18}$$

7.2.5 Optimization of Cost

The final equal to be used for optimization is:

$$C = C_M + C_L \tag{7.19}$$

where C_M is described by Equation (7.14) and C_L by (7.17). The constraints used for optimization are shown in Table 7.25. The values included are the absolute values for the bounds.

Table 7.25: Upper and Lower Bounds for Tolerances

Parameter	Lower bound (cm)	Upper bound (cm)
Tol_{Ds}	0.1	0.4
Tol_{Dig}	0.1	0.6
Tol_{Dog}	0.1	0.8
Sq	0	0.18

The optimization is performed, minimizing the total cost *C*, under the constraints in Table 7.23. The obtained outputs for the tolerances are included in Table 7.26.

Table 7.26: Optimal Tolerance Values

Parameter	Tolerance values (cm)
Tol_{Ds}	0.1761
Tol_{Dig}	0.4795
Tol_{Dog}	0.8

The squeeze is 0.18, which is within the allowable constraints for proper functioning of the seal assembly.

7.2.6 Validation

7.2.6.1 Cross-Validation

The 3-fold cross-validation method, described in Section 6.3, is used to confirm the utility of CA in the application.

The cross-validation method is first applied to the proposed framework that utilizes conjoint analysis. The 24 combinations in Table 7.17 are divided into 3 test sets of 8 each. The remainder in the complete set after each test set is removed is the training set. These are:

Training set 1: Combinations 2, 4, 7, 11, 13, 15, 17and 22 are removed

Training set 2: Combinations 1, 5, 6, 10, 14, 20, 21 and 24 are removed

Training set 3: Combinations 3, 8, 9, 12, 16, 18, 19 and 23 are removed

The tolerance allocation framework in Chapter 6 is applied to each training set and the complete set. The RMS error for each training set is calculated using Equation (6.5) and the total error is obtained using Equation (6.6).

The same procedure is applied to the proposed framework that does not utilize conjoint analysis. The importance vector is determined by the percentage preferences of a

100 experts. Out of a 100 experts, 20% voted for factor DS, 20% for GS, 30% for MM and 30% for PA. The 100 experts are divided into 3 test sets: two of them with 33 experts and one with 34 experts.

Training set 1: 15 of those who voted for DS, 10 for GS and 8 for PA are removed

Training set 2: 5 of those who voted for DS, 25 for MM and 3 for PA are removed

Training set 3: 10 of those who voted for GS, 5 for MM and 19 for PA are removed

The tolerance allocation framework in Chapter 6 is applied to each training set and the complete set. The RMS error for each training set is calculated using Equation (6.5) and the total error is obtained using Equation (6.6). The cross-validation results are shown in Table 7.27.

Table 7.27: Cross-Validation of Tolerance Allocation Framework

	$RMS_{e,total}$
Tolerance Allocation with CA	0.19747
Tolerance allocations without CA	0.21698

There is a decrease of 10.25% in the total RMS error value when CA method is used for the framework. This demonstrates the need for the use of CA in determining tolerances for the seal assembly using this framework.

7.3 Power Generating Shock Absorber (PGSA)

In the following section, the design of a Power-Generated Shock Absorber (PGSA) is considered to show the applicability of the proposed method for practical engineering problems.

A conventional shock absorber dampens movement of the suspension to keep the tire firmly on the ground. The kinetic energy is converted into heat energy, which is absorbed by the oil in the shock absorber. A PGSA uses a Linear Motion Electromagnetic System (LMES) to convert the heat energy into electrical energy. The LMES consists of a shaft with a magnetic wire around it, stator coil windings and electric control system. The electric control system manages the output electric voltage. The bottom shaft of the PGSA is connected to the moving suspension. The motion of the suspension causes the shaft to move around the magnet, causing an electric voltage output. The electricity can be combined with other sources of energy and stored in the batteries of an electric or hybrid car. A depiction of the system is shown in Figure 7.7 [65].

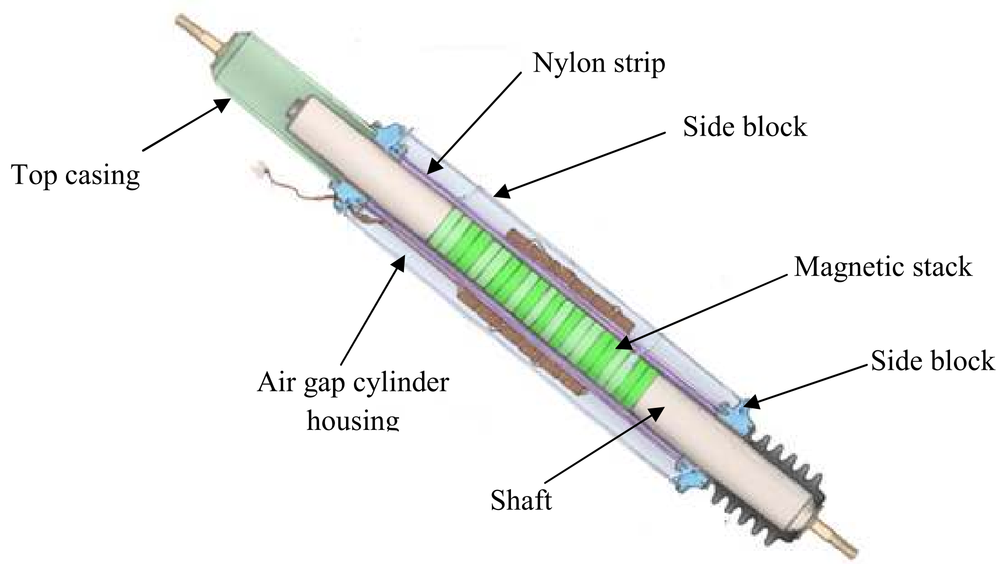


Figure 7.7: Power Generating Shock Absorber [65]

The performance of the vehicle depends on the proper functioning of the PGSA assembly. Improper tolerance allocation results in PGSA malfunctioning which is a detriment to passenger safety. Tolerance allocation determines the tolerances of the clearance locations, L_1 and L_2, in Figure 7.8. When the shaft of the PGSA is in motion, a huge gap in L_1/L_2 can cause improper connection with the wire and stator coil windings, resulting in failure to produce electricity. A huge interference in L_1/L_2 can hamper the motion of the shaft. This causes damage to the assembly, increases fatigue and reduces the lifecycle of the machine. Thus, it is important that the PGSA is toleranced tightly. However, if the tolerance is too tight, the cost of the component makes the product uncompetitive in the market. An optimal tolerance value is required to allow for proper functioning of the clutch while maintaining a competitive manufacturing cost.

The dimensioning used for tolerance allocation is shown in Figure 7.8 and the corresponding variables are listed in Table 7.28. The two air gaps are assumed to be constant (zero tolerance) and subtracted from the total thickness and housing length. To simplify calculation of the wire length, it is assumed to be wrapped around the shaft.

The proposed method, described in Chapter 6, is applied to the tolerance allocation problem of clutch assembly. The grades for each of the four factors to be used in this problem are summarized in Table 7.29.

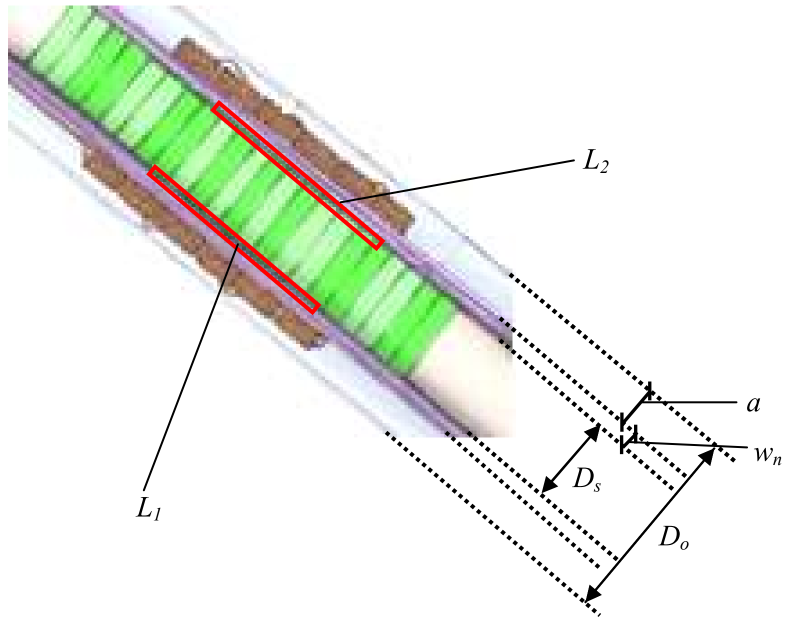


Figure 7.8 PGSA Dimensions

Table 7.28: Variable Descriptions and Dimensions for the PGSA

Variable	**Description**	**Value (cm)**
D_0	Overall diameter	60
a	Airgap	5
$D_{0a}= D_0\text{-}2a$	Overall Diameter minus airgap	50
D_s	Shaft Diameter	44
w_n	Width of nylon band	3

Table 7.29: Grade Divisions of Each Fuzzy Factor for PGSA

	Grade 1	**Grade 2**	**Grade 3**	**Grade 4**
U_1 (DS)	~0 cm	~20 cm	~40 cm	~60 cm
U_2 (GS)	Easy to manufacture	Hard to manufacture	-	-
U_3 (MM)	Poor	Medium	Good	-
U_4 (PA)	Poor	Medium	Good	-

7.3.1 Membership Degree Elicitation

To conduct the FCE process, the fuzzy subsets of the four factors, DS, GS, MM and PA (with fuzzy subsets U_1 to U_4) are determined first. The first factor, Dimension Size (DS), is the dimension of each part. The fuzzy subset for DS (U_1) is determined based on the value in Table 7.28 and listed in Table A.9 in the Appendix.

Membership degrees for GS, MM and PA are determined using the rank ordering method described in Chapter 3.2 Step 1. 100 experts are asked to compare two grades of each factor as to which one of them is better suited to describe the fuzzy factor for the part. This comparison is done for all combinations of grade pairs. A percentage preference is established for each grade which is the membership degree value of the grade for the assembly part.

For the fuzzy factor GS, the fuzzy subset (U_2) is listed in Table A.10 in the Appendix. The wire is hardest to machine due to its coiled shape. The outer casing is easiest to machine due to its cylindrical simplest geometry.

For the fuzzy factor MM, the fuzzy subset (U_3) is listed in Table A.11 as the Appendix. The experts determine the MM value based on the material malleability. A component, whose material has higher malleability, has higher MM. The magnetic wire is assumed to be of aluminum, with an iron shaft. A nylon band surrounds the wire. The outer casing is made of steel. Aluminum is the most malleable material while nylon is the least malleable [62]. The list of materials is shown in Table 7.30.

Table 7.30: PGSA Materials

Assembly part	**Material**
Shaft	Iron
Outer casing	Steel
Band	Nylon

For the fuzzy factor PA the fuzzy subset (U_4) is listed as Table A.12 in the Appendix. The experts rank order the subsets for each part based on how often the part is in contact with other parts of the system. The more contact that occurs for a part while engaging and disengaging the clutch, the higher the required PA is for proper functioning of the system. For this reason, the wire requires higher PA than the outer casing.

Once the fuzzy subsets are determined, the next step is to obtain the 1st order FCE matrix, shown in Equation 3.4. The FCE matrices for each of the factors are evaluated by experts as described for the clutch application in Section 7.1.1. The four matrices R_{DS}, R_{GS}, R_{MM} and R_{PA} are the FCE matrices for the factors GS, MM and PA respectively.

$$R_{DS}=\begin{bmatrix} 1 & 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0.5 & 1 & 0.5 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0.8 & 1 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.8 & 1 \end{bmatrix}$$

$$R_{GS}=\begin{bmatrix} 0 & 0.5 & 0.8 & 0.9 & 1 & 0.9 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0.7 & 1 & 0.8 & 0.5 & 0 \end{bmatrix}$$

$$R_{MM}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 & 1 & 0.8 \\ 0 & 0 & 0 & 0 & 0.7 & 1 & 0.7 & 0.05 & 0 & 0 \\ 1 & 0.3 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{and} \quad R_{PA}=\begin{bmatrix} 1 & 0.8 & 0.5 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 1 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0.8 & 1 \end{bmatrix}$$

7.3.2 Determining Importance Factors Using Conjoint Analysis

The CA method, described in Chapter 4, is applied to evaluate the weighted importance vector of fuzzy factors, *I*. The four grades, DS, GS, MM, and PA are the four attributes and they have four, two, three and three levels respectively. Thus, there are 72 combinations. To make ranking easier for the experts, a 1/3 fractional factorial is taken, resulting in a total of 24 combinations [45]. A fractional factorial design will take an adequate fraction of combinations, with as little an effect on the final combination as possible. The set of 24 combinations that are used for the Conjoint Analysis are shown in Table 7.31.

Table 7.31: Fractional Factorial Design for PGSA Application

Combination	***DS***	***GS***	***MM***	***PA***
1	~0 cm	Easy To Manufacture	Poor	Poor
2	~0 cm	Easy To Manufacture	Poor	Medium
3	~0 cm	Easy To Manufacture	Medium	Good
4	~0 cm	Hard to manufacture	Poor	Medium
5	~0 cm	Hard to manufacture	Medium	Poor
6	~0 cm	Hard To Manufacture	Medium	Medium
7	~20 cm	Easy To Manufacture	Poor	Poor
8	~20 cm	Easy To Manufacture	Poor	Good
9	~20 cm	Easy To Manufacture	Good	Poor
10	~20 cm	Hard to manufacture	Poor	Medium
11	~20 cm	Hard to manufacture	Poor	Good
12	~20 cm	Hard To Manufacture	Good	Medium
13	~20 cm	Hard to manufacture	Good	Poor
14	~40 cm	Easy to manufacture	Poor	Medium
15	~40 cm	Easy To Manufacture	Medium	Good
16	~40 cm	Easy To Manufacture	Good	Medium
17	~40 cm	Hard to manufacture	Medium	Medium
18	~40 cm	Hard to manufacture	Good	Good
19	~60 cm	Easy to manufacture	Poor	Poor
20	~60 cm	Easy To Manufacture	Medium	Poor
21	~60 cm	Hard to manufacture	Poor	Good
22	~60 cm	Hard to manufacture	Poor	Medium
23	~60 cm	Hard To Manufacture	Medium	Medium
24	~60 cm	Hard to manufacture	Good	Poor

The next step is to gain the respondent's preferences for each of the above combinations through customer surveys, computer programs, or elicitations from designers. The respondent for this application is an expert in the field of manufacturing the friction clutch. The subjective data elicited are based on the attributes and preferences on the respondent. Therefore the suggested final design will represent the preferred design as pertains to the expert(s) giving the rating/ranking data. This is shown in Table 7.32.

Table 7.32: Expert Rating of Attribute Combinations

Comb.	***DS***	***GS***	***MM***	***PA***	***Rank***
1	~0 cm	Easy To Manufacture(E)	Poor(P)	Poor(P)	4
2	~0 cm	Easy To Manufacture(E)	Poor(P)	Medium(M)	13
3	~0 cm	Easy To Manufacture(E)	Medium(M)	Good(G)	24
4	~0 cm	Hard to manufacture(H)	Poor(P)	Medium(M)	10
5	~0 cm	Hard to manufacture(H)	Medium(M)	Poor(P)	3
6	~0 cm	Hard to manufacture(H)	Medium(M)	Medium(M)	12
7	~20 cm	Easy To Manufacture(E)	Poor(P)	Poor(P)	2
8	~20 cm	Easy To Manufacture(E)	Poor(P)	Good(G)	22
9	~20 cm	Easy To Manufacture(E)	Good(G)	Poor(P)	8
10	~20 cm	Hard to manufacture(H)	Poor(P)	Medium(M)	9
11	~20 cm	Hard to manufacture(H)	Poor(P)	Good(G)	19
12	~20 cm	Hard to manufacture(H)	Good(G)	Medium(M)	17
13	~20 cm	Hard to manufacture(H)	Good(G)	Good(G)	23
14	~40 cm	Easy To Manufacture(E)	Poor(P)	Medium(M)	11
15	~40 cm	Easy To Manufacture(E)	Medium(M)	Medium(M)	14
16	~40 cm	Easy To Manufacture(E)	Good(G)	Poor(P)	7
17	~40 cm	Hard to manufacture(H)	Medium(M)	Good(G)	20
18	~40 cm	Hard to manufacture(H)	Good(G)	Medium(M)	16
19	~60 cm	Easy To Manufacture(E)	Poor(P)	Poor(P)	1
20	~60 cm	Easy To Manufacture(E)	Medium(M)	Good(G)	21
21	~60 cm	Hard to manufacture(H)	Poor(P)	Medium(M)	6
22	~60 cm	Hard to manufacture(H)	Poor(P)	Good(G)	18
23	~60 cm	Hard to manufacture(H)	Medium(M)	Poor(P)	5
24	~60 cm	Hard to manufacture(H)	Good(G)	Medium(M)	15

The Dummy Variable Regression table is shown in Table 7.33. The y-values are calculated using Logit coding in Equation (4.4-4.5).

Table 7.33: Dummy-Variable Binary Representation

Comb.	***DS(~cm)***				***GS***		***MM***			***PA***			***Rank***	***y***
	0	20	40	60	E	H	P	M	G	P	M	G		
1	1	0	0	0	1	0	1	0	0	1	0	0	21	-1.6582
2	1	0	0	0	1	0	1	0	0	0	1	0	22	0.08004
3	1	0	0	0	1	0	0	1	0	0	0	1	24	3.17805
4	1	0	0	0	0	1	1	0	0	0	1	0	16	-0.4055
5	1	0	0	0	0	1	0	1	0	1	0	0	12	-1.9924
6	1	0	0	0	0	1	0	1	0	0	1	0	11	-0.08
7	0	1	0	0	1	0	1	0	0	1	0	0	20	-2.4423
8	0	1	0	0	1	0	1	0	0	0	0	1	6	1.99243
9	0	1	0	0	1	0	0	0	1	1	0	0	8	-0.7538
10	0	1	0	0	0	1	1	0	0	0	1	0	10	-0.5754
11	0	1	0	0	0	1	1	0	0	0	0	1	18	1.15268
12	0	1	0	0	0	1	0	0	1	0	1	0	17	0.75377
13	0	1	0	0	0	1	0	0	1	0	0	1	4	2.44235
14	0	0	1	0	1	0	1	0	0	0	1	0	2	-0.2412
15	0	0	1	0	1	0	0	1	0	0	1	0	23	0.24116
16	0	0	1	0	1	0	0	0	1	1	0	0	3	-0.9445
17	0	0	1	0	0	1	0	1	0	0	0	1	5	1.38629
18	0	0	1	0	0	1	0	0	1	0	1	0	19	0.57536
19	0	0	0	1	1	0	1	0	0	1	0	0	13	-3.1781
20	0	0	0	1	1	0	0	1	0	0	0	1	15	1.65823
21	0	0	0	1	0	1	1	0	0	0	1	0	9	-1.1527
22	0	0	0	1	0	1	1	0	0	0	0	1	7	0.94446
23	0	0	0	1	0	1	0	1	0	1	0	0	14	-1.3863
24	0	0	0	1	0	1	0	0	1	0	1	0	1	0.40547

Linear dependency exists in the binary coding problem. In order to account for it, the binary matrix is modified by choosing a reference level for the part-worth utilites are based on. Effects coding is used to modify the matrix, which sets a reference level at '-1'. The reference attributes are chosen are DS of ~0 cm, GS of Easy to manufacture, MM of Poor and PA of Poor. The modified table is displayed in Table 7.34.

Table 7.34: Dummy-Variable Binary Representation

Comb.	***DS(~cm)***				***GS***		***MM***			***PA***			***Rank***	***Y***
	0	20	40	60	E	H	P	M	G	P	M	G		
1	-1	0	0	0	-1	0	-1	0	0	-1	0	0	21	1.658228
2	-1	0	0	0	-1	0	-1	0	0	-1	1	0	22	1.992430
3	-1	0	0	0	-1	0	-1	1	0	-1	0	1	24	3.178053
4	-1	0	0	0	-1	1	-1	0	0	-1	1	0	16	0.575364
5	-1	0	0	0	-1	1	-1	1	0	-1	0	0	12	-0.08004
6	-1	0	0	0	-1	1	-1	1	0	-1	1	0	11	-0.24116
7	-1	1	0	0	-1	0	-1	0	0	-1	0	0	20	1.386294
8	-1	1	0	0	-1	0	-1	0	0	-1	0	1	6	-1.15268
9	-1	1	0	0	-1	0	-1	0	1	-1	0	0	8	-0.75377
10	-1	1	0	0	-1	1	-1	0	0	-1	1	0	10	-0.40546
11	-1	1	0	0	-1	1	-1	0	0	-1	0	1	18	0.944461
12	-1	1	0	0	-1	1	-1	0	1	-1	1	0	17	0.753772
13	-1	1	0	0	-1	1	-1	0	1	-1	0	1	4	-1.65823
14	-1	0	1	0	-1	0	-1	0	0	-1	1	0	2	-2.44235
15	-1	0	1	0	-1	0	-1	1	0	-1	1	0	23	2.442347
16	-1	0	1	0	-1	0	-1	0	1	-1	0	0	3	-1.99243
17	-1	0	1	0	-1	1	-1	1	0	-1	0	1	5	-1.38629
18	-1	0	1	0	-1	1	-1	0	1	-1	1	0	19	1.15268
19	-1	0	0	1	-1	0	-1	0	0	-1	0	0	13	0.080043
20	-1	0	0	1	-1	0	-1	1	0	-1	0	1	15	0.405465
21	-1	0	0	1	-1	1	-1	0	0	-1	1	0	9	-0.57536
22	-1	0	0	1	-1	1	-1	0	0	-1	0	1	7	-0.94446
23	-1	0	0	1	-1	1	-1	1	0	-1	0	0	14	0.241162
24	-1	0	0	1	-1	1	-1	0	1	-1	1	0	1	-3.17805

A Regression Analysis is conducted using ANOVA Regression tool in Excel as described by Equation (4.3). In order to calculate the part-wroth utilities for the levels of each attribute, the regression model is solved with the variables in the binary matrix being the independent variables and the logit recoded rankings(*y*) being the dependent variables. The results of the Regression are displayed in Figure 7.9.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.978998084
R Square	0.958437248
Adjusted R Square	0.66960378
Standard Error	0.394301694
Observations	24

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	12	53.77840681	4.481534	43.23751	1.7518E-07
Residual	15	2.332107387	0.155474		
Total	27	56.1105142			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-1.889967489	0.233381273	-8.0982	7.41E-07	-2.3874079	-1.392527	-2.38740789	-1.3925271
X Variable 1	0	0	65535	#NUM!	0	0	0	0
X Variable 2	-0.301755911	0.268350167	-1.12449	0.27848	-0.8737308	0.2702189	-0.87373075	0.27021893
X Variable 3	-0.526882871	0.258861156	-2.03539	0.059877	-1.0786324	0.0248666	-1.07863236	0.02486662
X Variable 4	-0.661600462	0.240066806	-2.7559	0.014711	-1.1732907	-0.14991	-1.17329074	-0.1499102
X Variable 5	0	0	65535	#NUM!	0	0	0	0
X Variable 6	-0.485430428	0.185736925	-2.61354	0.019564	-0.8813193	-0.089542	-0.88131931	-0.0895415
X Variable 7	0	0	65535	#NUM!	0	0	0	0
X Variable 8	0.714786388	0.220771303	3.237678	0.005521	0.2442235	1.1853493	0.2442235	1.18534928
X Variable 9	1.434534621	0.222329114	6.452302	1.09E-05	0.96065134	1.9084179	0.96065134	1.90841791
X Variable 10	0	0	65535	#NUM!	0	0	0	0
X Variable 11	1.96729649	0.219842915	8.948646	2.11E-07	1.49871241	2.4358806	1.49871241	2.43588057
X Variable 12	3.871778067	0.22711208	17.04787	3.15E-11	3.38770013	4.355856	3.38770013	4.35585601

Figure 7.9: ANOVA Regression Results for PGSA example

The results reveal a good correlation between the logit recoded y-values and the dummy variables. The intercepts are the part-worth utilities for the levels of each attribute. The part-worths for the reference values are seen to be zero and the values for the other intercepts correspond to these levels. The part-worths are scaled to be positive by adding the minimum utility from a specified attribute to all other levels for that attribute.

Once the part-worth utilities are obtained for each level of each attribute, the importance of each attribute is calculated. The importance value is the difference an attribute contributes to the total utility of a product. As shown in Table 7.35, the difference is the range in the utilities for each attribute. Percentage values, that add up to 100%, are calculated from the ranges. For the given application, DS has an importance of 10.25%, GS has an importance of 7.52%, MM of 22.22% and PA of 60.01%.

Table 7.35: Calculation of Attribute Importance

Attributes	*Levels*	*Part-Worth Utilities*	*Range*	*Attribute Importance*
DS	~0 cm	0.6616	0.6616-0=0.6616	(0.6616/6.4533)*100=**10.25%**
	~20 cm	0.4853		
	~40 cm	1.4354		
	~60 cm	0		
GS	Easy(E)	0.4853	0.4853-0=0.4853	(0.4854/6.4533)*100=**7.52%**
	Hard(H)	0		
MM	Poor	0	1.4345-0=1.4345	(1.4345/6.4533)*100=**22.22%**
	Medium	0.7148		
	Good	1.4345		
PA	Poor	0	3.8718-0=3.8718	(0.6616/6.4533)*100=**60.01%**
	Medium	1.9673		
	Good	3.8718		
Total			6.4533	

7.3.3 Tolerance allocation Using FCE

The machinability values are determined for the parts using the FCE method in Chapter 3 and displayed in Table 7.36.

Table 7.36: Machinability Values for Parameters

Parameter	Machinability ζ
D_{0a}	0.4073
D_s	0.6606
w_n	0.6377

In an ideal PGSA that does not take tolerances into account, the equations for direction of horizontal motion, based on the dimensions in Figure 7.8, is:

$$D_{0a} - D_s - 2w_n = 0 \tag{7.20}$$

Once the tolerances are added into the equation, Equation (7.21) now becomes:

$$(D_{0a} + Tol_{D_{0a}}) - (D_s + Tol_{D_s}) - 2(w_n + Tol_{w_n}) = 2Tol_{L1} = 2Tol_{L2} \tag{7.21}$$

where Tol_{L1} and Tol_{L2} are the tolerances at locations L_1 and L_2 respectively, in Figure 7.8. Tol_i is the tolerance of part i in the assembly. After combining Equations (7.21) and (7.22), one obtains:

$$\frac{Tol_{D_{0a}}}{2} - \frac{Tol_{D_s}}{2} - Tol_{w_n} = Tol_{L1} = Tol_{L2} \tag{7.22}$$

Equation (7.22) is the assembly function equation that represents Equation (3.14) in Chapter 3. The coefficients of the tolerances in these equations are the values of the sensitivity coefficient, *ξ*, in the equation. The comprehensive factor, *Ψ*, is calculated using Equation (3.15). The values are listed in Table 7.37.

Table 7.37: Comprehensive Factor Values for Parameters

Parameter	Sensitivity coefficient ξ	Comprehensive factor Ψ
D_{0a}	1/2	1.6292
D_s	-1/2	2.6425
w_n	-1	0.6377

The total machining cost is obtained using Equation 3.16.

$$C_m = C_0 + \frac{1.6292}{Tol_{D_{0a}}} + \frac{2.6425}{Tol_{D_s}} + \frac{0.6377}{Tol_{w_n}} \tag{7.23}$$

Where C_m is the total cost of machining, and C_0 is the initial setup costs. C_0 is \$20 for this application.

7.3.4 Determining Costs due to Performance Variation

There are two factors that determine the performance of the PGSA, the vertical acceleration and the energy generated by the PGSA. The length of the wire, l_w, is an important design variable in the estimation of these two factors. The wire is assumed to be coiled around the shaft. The wire is coiled along the shaft for a length of 75 cm with the coils aligned perfectly perpendicular to the axis of the cylinder. The wire thickness is 0.5 cm. The number of coils in the wire is approximately 75/0.5=150. The total length of the wire, l_w, is:

$$l_w \approx 150\pi D_s \tag{7.24}$$

The tolerance of the wire length now becomes:

$$\Delta l_w \approx 150\pi \Delta D_s \tag{7.25}$$

To represent the relationship between each of the design variables and the performance factors, an algebraic model was formed using Dymola [66]. The model, shown in Figure 7.10 uses predefined relations to represent the spring force, mass, gravity, and electrical components. A linear motor model was created to incorporate the contribution of the magnetic strength and wire length on the vertical acceleration and the amount of generated energy. A detailed description of the PGSA and analytical equations are available in Refs. [65].

The amount of generated energy is calculated by integrating the power generated from the PGSA which is determined from Joule's Law. As can be seen from Figure 7.10 b, a model of the entire car was made with four PGSA suspension systems attached to four tire models, a mass to represent the weight of the car, and constant load acting on the mass to represent gravity. The total generated energy is determined to be a combination of the energy produced by all four PGSA's. The vertical acceleration is calculated by using a predefined accelerometer model in Dymola.

The vertical acceleration has a nominal value that indicates best output performance. A higher value will cause damage to the PGSA, while a lower value will lower production of electricity. Thus, a 'nominal is best' Taguchi loss function is used.

The generated energy uses a similar loss function. Too little generated energy does not validate use of the device in the car. Too much energy causes heating in the wires and damage to the electrical components.

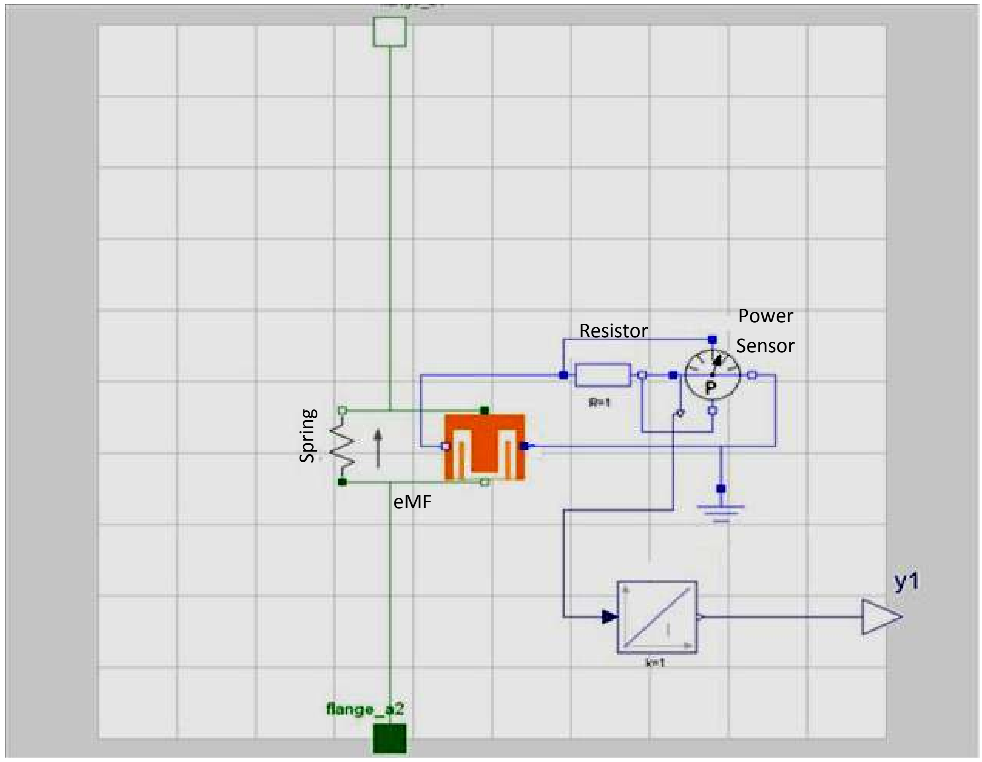


Figure 7.10 (a): Dymola Model for Suspension System of PGSA

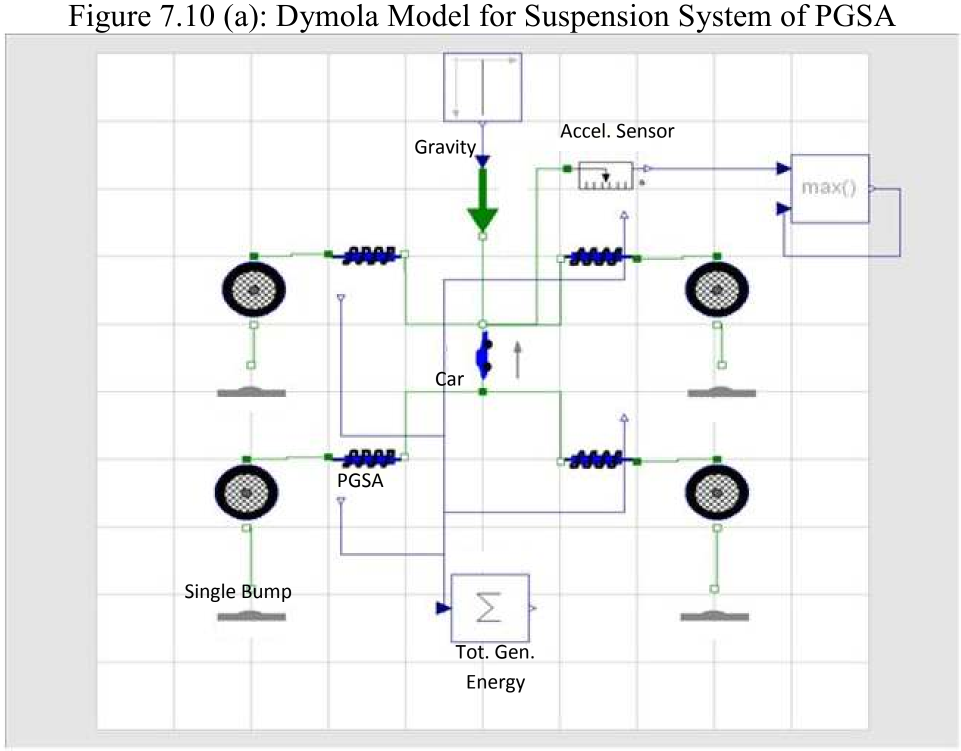


Figure 7.10: (b) Dymola Model of Full Car

The cost incurred due to loss in performance, C_L, is given by:

$$C_L = k_1(a_c - a_t)^2 + k_2(P - P_t)^2 \tag{7.26}$$

where k_1 and k_2 are the Taguchi loss constants. The loss constants, k_1 and k_2 are determined by the costs of replacing parts that deviate a certain amount from the mean.

If the vertical acceleration deviates from the target of 12.7 m/s^2 by 2 m/s^2, the PGSA shaft breaks down and the cost of replacement is $100.

$$100 = k_1(2)^2$$

$$k_1 = 25$$

If the same shaft overheats with an increase in energy of 10 J, then

$$100 = k_2(10)^2$$

$$k_2 = 1$$

Equation (7.26) now becomes

$$C_L = 25(a_c - a_t)^2 + (P - P_t)^2 \tag{7.27}$$

7.3.5 Optimization

The final equation to be used for optimization is:

$$C = C_M + C_L \tag{7.28}$$

where C_M is described by Equation (7.24) and C_L by (7.27). The constraints used for optimization are shown in Table 7.38. The values included are the absolute values for the bounds.

Table 7.38: Upper and Lower Bounds for Tolerances

Parameter	Lower bound (cm)	Upper bound (cm)
Tol_{D0a}	0.01	1
Tol_{Ds}	0.01	0.8
Tol_{wn}	0.01	0.2
Tol_{L1}	0.01	0.2
Tol_{L2}	0.01	0.2

The optimization is performed, minimizing the total cost *C*, under the constraints in Table 7.38.

The obtained outputs for the tolerances and the performance are included in Table 7.39.

Table 7.39: Optimal Tolerance Values

Parameter	Tolerance values (cm)
Tol_{D0a}	0.9039
Tol_{Ds}	0.1040
Tol_{wn}	0.2
Tol_{L1}	0.2
Tol_{L2}	0.2

The generated energy of the optimized PGSA is 253.35 J, which is a deviation of 1.4% from the targeted energy. The vertical acceleration of the PGSA is 12.7339 m/s^2, which is a deviation of 0.27% from the target acceleration.

Both locations L_1 and L_2 have a gap of 0.2 cm, which is within the allowable constraints for proper functioning of the PGSA.

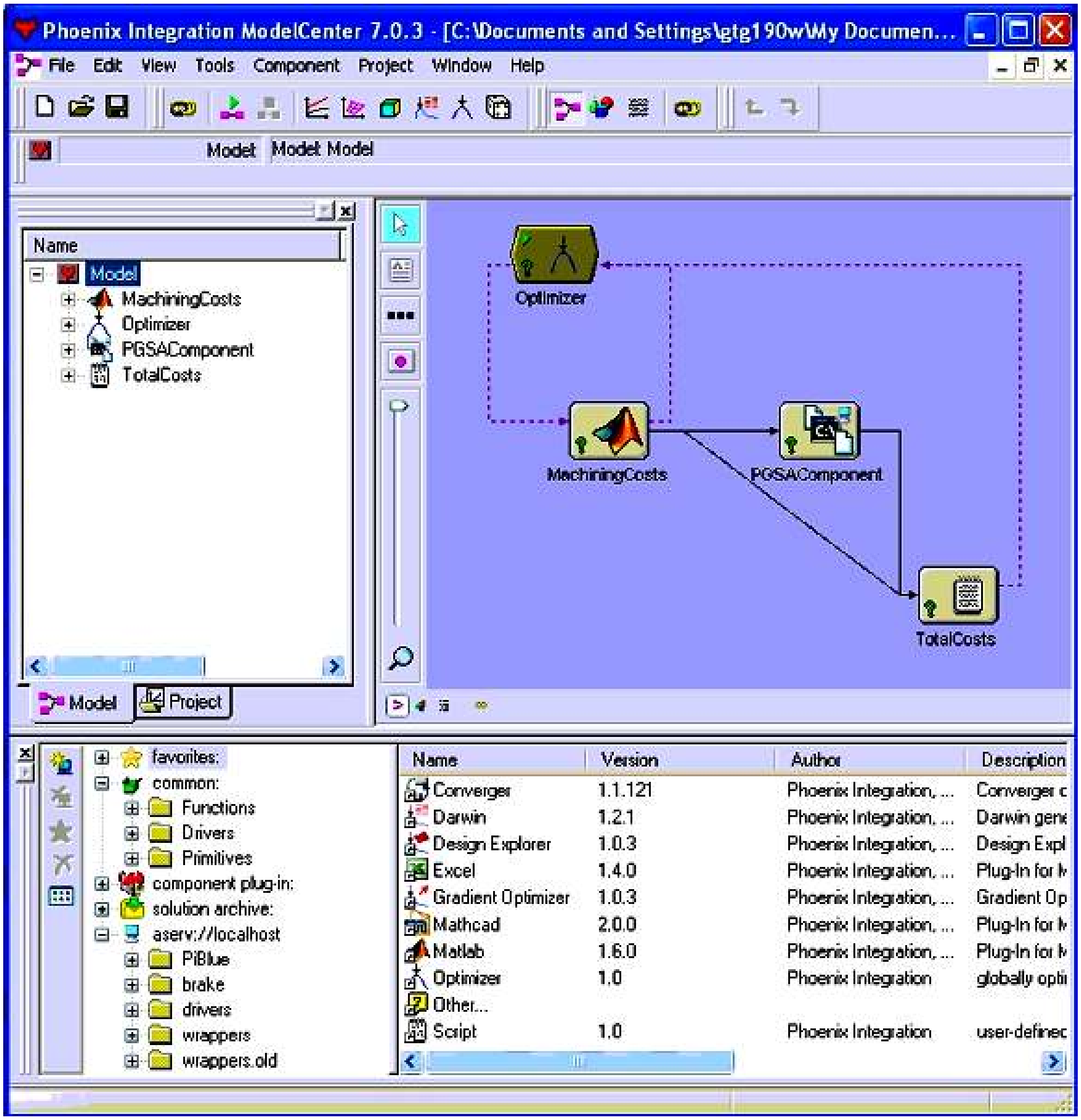


Figure 7.11: Optimization Component of PGSA example using ModelCenter

ModelCenter is used to run simulations of the PGSA model during the optimization of the tolerances. This is shown in Figure 7.11. The Matlbab component computes the machining costs using the FCE method. The PGSA computes the output performance. The output performance is used to calculate the quality loss costs in the script, which determines the total cost *C*. The total cost *C* is optimized with the tolerances as the design variables.

7.3.6 Validation

7.3.6.1 Cross-Validation

The 3-fold cross-validation method, described in Section 6.3, is used to confirm the utility of CA in the application.

The cross-validation method is first applied to the proposed framework that utilizes conjoint analysis. The 24 combinations in Table 7.31 are divided into 3 test sets of 8 each. The remainder in the complete set after each test set is removed is the training set. These are:

Training set 1: Combinations 3, 4, 9, 10, 13, 14, 18 and 22 are removed.

Training set 2: Combinations 1, 5, 7, 12, 15, 20, 21 and 23 are removed.

Training set 3: Combinations 2, 6, 8, 11, 16, 17, 19 and 24 are removed.

The tolerance allocation framework in Chapter 6 is applied to each training set and the complete set. The RMS error for each training set is calculated using Equation (6.5) and the total error is obtained using Equation (6.6).

The same procedure is applied to the proposed framework that does not utilize conjoint analysis. The importance vector is determined by the percentage preferences of a 100 experts. Out of a 100 experts, 10% voted for factor DS, 20% for GS, 30% for MM and 40% for PA. The 100 experts are divided into 3 test sets: two of them with 33 experts and one with 34 experts.

Training set 1: 10 of those who voted for DS, 8 for GS and 15 for PA are removed.

Training set 2: 10 of those who voted for GS, 10 for MM and 13 for PA are removed.

Training set 3: 2 of those who voted for GS, 20 for MM and 12 for PA are removed.

The tolerance allocation framework in Chapter 6 is applied to each training set and the complete set. The RMS error for each training set is calculated using Equation (6.5) and the total error is obtained using Equation (6.6). The cross-validation results are shown in Table 7.40.

Table 7.40: Cross-Validation of Tolerance Allocation Framework

	$RMS_{e,total}$
Tolerance Allocation with CA	0.00364
Tolerance allocations without CA	0.00787

There is a substantial increase in RMS error on removing the CA framework. This demonstrates that the need for conjoint analysis in the framework.

7.3.6.2 Validation of Taguchi's Loss function

The propose framework is applied with and without use of Taguchi's loss function. The deviation from the expected clutch torque capacity is noted and displayed in Table 7.41.

Table 7.41: Deviation in Performance with and without Taguchi's Quality Loss Function

	Energy(J)	*Deviation (%)*	a_c(m/s^2)	*Deviation (%)*	*Quality Loss Costs($)*
TA with Taguchi	253.35	1.4	12.7339	0.27	13.37
TA without Taguchi	231.06	10.09	12.8885	1.4	673.61

If the Taguchi quality loss function is not included, it is seen that there is a significant increase in deviation from the target performance for Energy generated. The deviation increases to 10.09%. The deviation in vertical acceleration increases to 1.4%. As a result, the use of Taguchi's loss function lowers the repair costs from $673.61 to $13.37. This demonstrates the effectiveness of the Taguchi's loss function in increasing the robustness of the procedure.

CHAPTER 8. CONCLUSION

8.1 Summary

The purpose of the current research is to develop a computationally efficient tolerance allocation method that takes into account the needs of both the designer and the manufacturer. The Fuzzy Comprehensive Evaluation procedure (FCE) allocates tolerances based on certain 'fuzzy' (i.e. subjective) factors that are considered significant by the manufacturer. The Taguchi quality loss function reduces variation in the final output performance of the assembly while allocating tolerances. Also, the current method of calculating weighted importance of fuzzy factors in FCE process is improved through use of the Conjoint Analysis (CA) procedure.

The proposed framework is applied to three engineering applications: Tolerance allocation of a friction clutch, of an accumulator O-ring seal and of a PGSA. Each of these applications undergoes a cross-validation process to determine the utility of Conjoint Analysis in the procedure. The costs in output performance deviation are also calculated to determine the costs saved due to use of Taguchi quality loss function.

For the clutch application, the variation in output clutch torque capacity is minimized using Taguchi's quality loss function. For the PGSA example, the shaft vertical acceleration and the energy generation are the output variables used for Taguchi's quality loss function.

The Cross-Validation procedure reveals that the CA procedure is more efficient in calculating tolerances in all of the three processes. In the preceding method, the experts needed to determine which attributes are most important or rate them. All of the attributes are considered important in the process, so it is difficult for the experts to determine which one is most important. It is easier for them to rank combination of levels of attributes, which is done in the CA procedure.

The use of Taguchi quality loss function produces significant savings in cost for the clutch and PGSA assembly.

8.2 Limitations

The research presents a novel approach to tolerance allocation using FCE, CA and Taguchi's quality loss function. Nevertheless, it has the following limitations:

1) The main problem with utilizing this Conjoint Analysis method (Conjoint Value Analysis) is that there is an increase in error as the number of fuzzy factors increases. As the number of factors increase, more combinations need to be ranked, which increases fatigue on the user. If there are more than six fuzzy factors, this method is not generally used [43].
2) The membership degrees for the fuzzy factors are elicited using pair wise comparisons of the fuzzy grades of each factor. If the number of grades for the factor is large, each expert has to do a large number of pair wise comparisons, which leads to increase in fatigue on the user.
3) The losses due to variation in output performance of the assembly are minimized in this framework. Each of the three applications mentioned also undergo repair costs due to fatigue, which are related to the tolerances.

8.3 Future Work

The current framework allocates tolerances with the objective of minimizing costs and decreasing variation in performance. The efficiency of the framework is shown with three engineering applications: a friction clutch assembly, an accumulator seal assembly and a PGSA assembly. However, there is still scope for improving the framework in the future.

1) The optimized results can be prototyped and tested to check the variation in output performance and calculate the assembly costs.
2) Research a framework to calculate the fatigue life as a function of the tolerances.
3) Incorporate a different Decision Support Process, such as Adaptive Conjoint Analysis [41] if there are more than six fuzzy factors.

APPENDIX A

Table A.1. Membership Degrees for Factor DS size of the Clutch Problem (U_1)

	Grade 1	**Grade 2**	**Grade 3**	**Grade 4**
r_f	0	0.68	0.32	0
r_{cr}	0.79	0.21	0	0
r_d	0	0.87	0.13	0
$r_{d,a}$	0.1	0.9	0	0
T_f	0.61	0.39	0	0
T_p	0.45	0.55	0	0
T_c	1	0	0	0
$T_{0,a}$	0	0.98	0.02	0
L_{hn}	0	0	0.5	0.5

Table A.2. Membership Degrees of Factor GS of the Clutch Problem (U_2)

	Grade 1	**Grade 2**
r_f	0.45	0.55
r_{cr}	1	0
r_d	0.85	0.2
$r_{d,a}$	0.85	0.2
T_f	0.45	0.55
T_p	0.15	0.9
T_c	0.85	0.2
$T_{0,a}$	0.15	0.9
L_{hn}	0.75	0.25

Table A.3. Membership Degrees of Factor MM for the Clutch Problem (U_3)

	Grade 1	**Grade 2**	**Grade 3**
r_f	0.8	0.1	0.1
r_{cr}	0.15	0.7	0.15
r_d	0	0.1	0.9
$r_{d,a}$	0	0.1	0.9
T_f	0.8	0.1	0.1
T_p	0.15	0.7	0.15
T_c	0	0.1	0.9
$T_{0,a}$	0.8	0.1	0.1
L_{hn}	0.8	0.1	0.1

Table A.4. Membership Degrees for Factor PA of the Clutch Problem (U_4)

	Grade 1	**Grade 2**	**Grade 3**
r_f	0.1	0.5	0.4
r_{cr}	0.6	0.4	0
r_d	0	0.2	0.8
$r_{d,a}$	0	0.2	0.8
T_f	0.1	0.5	0.4
T_p	0.1	0.8	0.2
T_c	0	0.2	0.8
$T_{0,a}$	0	0.2	0.8
L_{hn}	0.9	0.1	0.1

Table A.5. Membership Degrees for Factor DS of the O-ring Seal (U_1)

	Grade 1	**Grade 2**	**Grade 3**	**Grade 4**
D_s	0.5	0.5	0	0
D_{ig}	0	1	0	0
D_{og}	0	0	0	1

Table A.6. Membership Degrees of Factor GS of the O-ring seal (U_2)

	Grade 1	**Grade 2**
D_s	0.5	0.5
D_{ig}	0.1	0.9
D_{og}	0.7	0.3

Table A.7. Membership Degrees of Factor MM of the O-ring Seal (U_3)

	Grade 1	Grade 2	Grade 3
D_s	0.2	0.2	0.6
D_{ig}	0.25	0.5	0.25
D_{og}	0.7	0.15	0.15

Table A.8. Membership Degrees for Factor PA of the O-ring seal (U_4)

	Grade 1	Grade 2	Grade 3
D_s	0.1	0.1	0.8
D_{ig}	0.2	0.5	0.3
D_{og}	0.4	0.4	0.2

Table A.9: Membership Degrees for Factor DS for the PGSA Example (U_1)

	Grade 1	Grade 2	Grade 3	Grade 4
D_{oa}	0	0	0.5	0.5
D_s	0	0	0.3	0.7
w_n	0.85	0.15	0	0

Table A.10: Membership Degrees of Factor GS for the PGSA Example (U_2)

	Grade 1	Grade 2
D_{oa}	0.6	0.4
D_s	0.5	0.5
w_n	0.3	0.7

Table A.11: Membership Degrees of Factor MM for the PGSA Example (U_3)

	Grade 1	**Grade 2**	**Grade 3**
D_{oa}	0	0.2	0.8
D_s	0	0.5	0.5
w_n	0.8	0.2	0

Table A.12: Membership Degrees for Factor PA of the O-ring Seal (U_4)

	Grade 1	**Grade 2**	**Grade 3**
D_{oa}	0.6	0.4	0
D_s	0.2	0.1	0.7
w_n	0.3	0.2	0.5

REFERENCES

1. Tewis, J. and Perryman, S., *J.D. Power and Associates Reports: Embattled U.S. Automakers Make Substantial Gains in Initial Quality, Outpacing Industry-Wide Improvement*. 2009.
2. Barth, L., *2009 Annual Auto Reliability Survet: Domestics vs. the Imports*. 2009.
3. Stewart, M., *Variation Simulation of Fixed Assembly Processes for compliant structures using piecewise-linear analysis*, in *Brigham Young University*. 2004.
4. K. W. Chase, A.R.P., *A Survey of Research in the Application of Tolerance Analysis to the Design of Mechanical Assemblies*, in *Mechanical Engineering Department*. 1991, Brigham Young University: Provo, Utah.
5. Placek, C., 1989. *Mechanical Tolerancing Workshop.* Quality World Technical Supplement: pp. 16-17.
6. Ji, S., Li, X. and Cai, H., 2000. *Optimal Tolerance Allocation Based on Fuzzy Comprehensive Evaluation and Genetic Algorithm.* Manufacturing Review, **1** (1): pp. 50-59.
7. Ross, T.J., *Fuzzy Logic With Engineering Applications*. 2nd ed. 2004, John Wiley & Sons, Ltd.
8. Chase, K.W., *Tolerance Allocation Methods for Designers*, in *Brigham Young University*. 1999.
9. K. W. Chase, W.H.G., B. G. Loosli, L.F. Hauglund, 1990. *Least Cost Tolerance Allocation for Mechanical Assemblies with Automated Process Selection.* Manufacturing Review, **3** (1): pp. 49-59.
10. Chase, K.W. and Greenwood, W.H., 1998. *Design Issues in Mechanical Tolerance Analysis.* Manufacturing Review, **1** (1): pp. 50-59.
11. Fortini, E.T., *Dimensioning for Interchangeable Manufacture*. 1967, New York, Industrial Press.
12. Weiss, P., *A Discussion of Scientific Methods for Setting Manufacturing Tolerances*. 1993, University of Wisconsin: Masison, Wisconsin.

13. Chase, K.W., *Minimum-Cost Tolerance Allocation*. 1999, Brigham Young University: Provo, Utah.

14. Edel, D.H. and Auer, T.B., 1965. *Determine the Least Cost Combination for Tolerance Accumulations in a Drive Shaft Seal Assembly.* General Motors Engineering Journal: pp. 36-38.

15. Wilde, D. and Prentice, E., 1975. *Minimum Exponential Cost Allocation of Sure-Fit Tolerances.* ASME, **97**: pp. 1395-1398.

16. Speckhart, F.H., 1972. *Calculation of tolerance based on a minimum cost approach.* Journal of Engineering for Industry, **5**: pp. 447-453.

17. Peters, J., 1970. *Tolerancing the Components of an Assembly for Minimum Cost.* Journal of Engineering for Industry: pp. 677-682.

18. Z. Dong, W.H., 1991. *Optimal process sequence identification and optimal process tolerance assignement in computer-aided process planning.* Computers in Industry, **17**: pp. 19-32.

19. Z. Dong, W.H., D. Xue, 1994. *New production costs-tolerance models for tolerance synthesis.* Journal of Engineering for Industry, **116**: pp. 199-206.

20. Greenwood, W.H. and Chase, K.W., 1988. *Worst Case Tolerance Analysis with Nonlinear Problems.* Journal of Engineering for Industry, **ASME** (112): pp. 382-384.

21. Parkinson, D.B., 1985. *Assessment and Optimization of Dimensional Tolerances.* Computer-Aided Design, **17** (4): pp. 191-199.

22. Spotts, M.F., 1973. *Allocation of tolerance to minimize costs of assembly.* Journal of Engineering for Industry: pp. 762-764.

23. Sutherland, G.H. and Roth, B., 1975. *Mechanism design: accounting for machining tolerances and costs in function generating problems.* Journal of Engineering for Industry, **2**: pp. 283-286.

24. Lee, W. and Woo, T.C., 1990. *Tolerances: Their Analysis and Synthesis.* Journal of Engineering for Industry, **112**: pp. 113-121.

25. Bennett, G. and Gupta, L.C., 1969. *Least Cost Tolerances-I and II.* International Journal of Production Research, **8**: pp. 65-74, 169-181.

26. Andersen, C.B., *General System for Least Cost Tolerance Allocation in Mechanical Assemblies*. 1990, Brigham Young University.
27. Dong, Z. and Soom, A., 1990. *Automatic optimal tolerance design for related dimension chains.* ASME Machining Review, **3** (4): pp. 262-271.
28. Michael, W. and Siddall, J.N., 1981. *Optimization problem with tolerance assignment and full acceptance.* ASME Journal of Mechanical Design, **103**: pp. 842-848.
29. Lee, W.J. and Woo, T.C., 1989. *Optimum selection of discrete tolerances.* Journal of Mechanisms, Transmissions and Automation in Design, **111**: pp. 243-251.
30. Du, L., Choi, K.K. and Youn, B.D., *A New Fuzzy Analysis Method for Possibility-based Design Optimization*, in *10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*. 2004: Albany, New York.
31. Qinghong, C., 1994. *Fuzzy Comprehensive evaluation of design variables in mechanical reliability.* Fuzzy Techniques and Application: pp. 492-497.
32. *J. D. Power: Decision Support Systems Glossary*. 1999 [cited 2010 03/10]; Available from: http://DSSResources.COM/glossary/.
33. Bohanec, M., *What is Decision Support?*, Jozef Stefan Institute.
34. Keen, P. and Morton, M.S., *Decision Support Systems: An organizational Perspective*. 1978, Reading, MA, Addison Wesley Publishing.
35. Shim, J.P., et al., 2002. *Past, present and future of decision support technology.* Decision Support Systems, **33**: pp. 111-126.
36. Pearson, J.M. and Shim, J.P., 1995. *An empirical investigation into DSS structures and environments.* Decision Support Systems, **13**: pp. 141-158.
37. Nolan, R.L. and Croson, D.C., *Creative Destruction*. 1995, Boston, MA, Harvard Business School Press.
38. Hazelrigg, G.A., 2003. *Validation of Engineering Design Alternative Selection Methods.* Engineering Optimization, **35** (2): pp. 103-120.
39. Decker, M.M.a.R., *An Empirical Comparison of CBC and AHP for Measuring Consumer Preferences*, Bielefeld University: Bielefeld, Germany.
40. *Cva: A Full-Profile Conjoint Analysis System from Sawtooth Software*. 2002, Sawtooth Software: Sequim, Washington.

41. *Aca System for Adaptive Conjoint Analysis*, in *Technical Report No. Sawtooth Software, Inc*. 2007, Sawtooth Software: Sequim, Washington.

42. *The Cbc System for Choice-Based Conjoint Analysis*, in *Technical Report No. Sawtooth Software, Inc*. 2008: Sequim, Washington.

43. Hjelkrem, C., 2008. *Market Scenarios and Conjoint Analysis.* Telektronikk, **104** (3/4): pp. 136-143.

44. Box, G.E., Hunter, W.G. and Hunter, J.S., *Statistics for Experimenters: Design, Innovation and Discovery*, in *Wiley*. 2005.

45. McKay, M.D., Beckman, R.J. and Conover, W.J., 1979. *A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output Form a Computer Code.* Technometrics, **21** (2): pp. 239-245.

46. Amarchinta, H. and Grandhi, R.V., 2008. *Multi-Attribute Structural Optimization Based on Conjoint Analysis.* AIAA Journal, **46** (4): pp. 884-893.

47. Bech, M. and Hansen, D.G., 2006. *Effects Coding in Discrete Choice Experiments.* Health Economics, **14** (10): pp. 1079-1083.

48. Adamowicz, W., Louviere, J. and Williams, M., 1994. *Combining Revealed and Stated Preference Methods for Value Environmental Amenities.* Journal of Environmental Economics and Management, **26** (3): pp. 271-292.

49. Orme, B.K., *Getting Started With Conjoint Analysis: Strategies for Product Design and Pricing Research*. 2006, Wisconsin, Research Publishers LLC.

50. Bretscher, O., *Linear Algebra With Applications*. 3rd ed. 1995, Upper Saddle River, NJ, Prentice Hall.

51. Wolberg, J., *Data Analysis Using the Method of Least Squares: Extracting the Most Information from Experiments*. 2005, Springer.

52. Sawtooth Software, *CVA: A Full-Profile Conjoint Analysis System from Sawtooth Software*, in *Technical Paper Series*. 2002, Sawtooth Software, Inc: Sequim, Washington.

53. Antony, J., 1997. *Experiments in quality.* Manufacturing Engineer, **76** (6): pp. 272-275.

54. Ross, P.J., *Taguchi techniques for quality engineering*. 1995, New York, NY, McGraw-Hill.

55. Yacout, S. and Boudreau, J., 1998. *Assessment of quality activities using Taguchi's loss function.* Computers and Industrial Engineering, **35** (1-2): pp. 229-232.
56. Albright, T.L. and Roth, H.P., 1994. *Managing quality through quality loss function.* Journal of Cost Management: pp. 20-37.
57. Quinlan, J., *Process Improvement by the application of Taguchi methods*, in *Third Taguchi Symposium*. 1985: Michigan. p. 11-16.
58. Ramburg, J.S., *Improvements on Taguchi methods for semiconductor design/manufacture*, in *Semiconductor Manufacturing Conference*. 1989: Phoenix, Arizona.
59. Pereira, Z.L. and Aspinwall, E., 1993. *Off-line quality control applied to food products.* Total Quality Management Journal, **4** (1): pp. 57-69.
60. Anthony, J. and Kaye, M., 1996. *Optimization of core tube life using Taguchi experimental design methodology.* Quality World Technical Supplement: pp. 42-50.
61. Kohavi, R. *A Study of Cross-Validation and Bootstrap for Accuracy Estimation and Model Selection*. in *International Joint Conference on Artificial Intelligence*. 1995.
62. Rich, J.C., *The Materials and Method of Sculpture*. 1988, Courier Dover Publications.
63. *11 Basic Tech Questions On...Putting the Power to the Ground*, in *Turbo and High Tech Performance*. 2009.
64. Corporation, P.H., *A Series Piston Accumulators*. 2004: Watford, Herts, UK.
65. Community, P. *Power Generating Shock Absorber*. 2005 [cited 2010 04/02]; Available from: http://www.peswiki.com/index.php/Directory:Power-Generating_Shock_Absorber.
66. *Dynamic Modeling Laboratory*, in *Dymola user's Manual*. 2002, Dynasim AB: Sweden.